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A CASE STUDY

Mathematics in ancient India

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Abstract

"India was the motherland of our race and Sanskrit the mother of Europe's languages. India was the mother of our philosophy, of much of our mathematics, of the ideals embodied in Christianity... of self-government and democracy. In many ways, Mother India is the mother of us all."- Will Durant (American Historian 1885-1981). Mathematics had and is playing a very significant role in development of Indian culture and tradition. Mathematics in Ancient India or what we call it Vedic Mathematics had begun early Iron Age. The SHATAPATA BRAHAMANA AND THE SULABHSUTRAS includes concepts of irrational numbers, prime numbers, rule of three and square roots and had also solved many complex problems. Ancient Hindus have made tremendous contributions to the world of civilization in various fields such as Mathematics, Medicine, Astronomy, Navigation, Botany, Metallurgy, Civil Engineering, and Science of consciousness which includes Psychology and philosophy of Yoga. The object of this paper tries is to make society aware about the glorious past of our ancient science which has been until now being neglected without any fault of it.

Key Words : Ancient, Mathematics, *Ganita*, *Sulabhsutras*

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Any account of the classical sciences of India has to begin with mathematics, the reasons being, the ancient Sanskrit text Vedanga Jyotisa (ca. fourth century B.C.E.) says,

Like the crest on the peacock's head, Like the gem in the cobra's hood so stands mathematics at the head of all the sciences.

The Sanskrit word used for mathematics in this verse is *ganita*, which literally means "reckoning."

The unique Indian view about the classical of mathematics is that number was treated as the primary concept.

This thing has been substantiated by a distinguished Swiss mathematician-physicist in 1929 that when he wrote:-

"Occidental mathematics has in past centuries broken away from the Greek view and followed a course which seems to have originated in India"

The love affair between Indian culture and numbers has been long. When most societies had difficulty in handling numbers beyond 1000, the Buddhist text Lalita-vistara (before the fourth century C.E.) not only has no problem with huge numbers, but seems to revel in giving them names (10145, the highest number quoted, being called dhvaja-nis'a-mani.)

Indian mathematics which we call today Ancient Indian Mathematics has it's origination in Indian subcontinent from 1200 BCE upto the end of the 18th century. In this classical period of Indian mathematics (400 CE to 1600 CE), not only important but excellent contributions were made by scholars such as Aryabhata, Brahmagupta, Mahâvîra, Bhaskara II, Madhava of Sangamagrama and Nilakantha Somayaji.

The decimal number system which we use today was first recorded in Indian mathematics. Indian mathematicians made early contributions to the study of the concept of zero as a number, negative numbers, arithmetic, and algebra not only this but In addition, trigonometry was further advanced in India and in particular, the modern definitions of sine and cosine were developed here also in India.

These mathematical concepts were transmitted to the Middle East, China, and Europe and led to further developments that now form the foundations of many areas of mathematics.

Ancient and medieval Indian mathematical works, was composed in Sanskrit, which usually consisted of a section of *sutras* in which a set of rules or problems were stated with great beauty in verse in order to aid memorization by a student. This was followed by a second section consisting of a prose commentary (sometimes multiple commentaries by different scholars) that explained the problem in more detail and provided justification for the solution. In the prose section, the form (and therefore, its memorization) was not considered as important as the ideas involved.

All mathematical works were transmitted from one generation to other orally until approximately 500 BCE; thereafter, they were transmitted both orally and in manuscript form. The oldest extant mathematical *document* produced on the Indian subcontinent is the birch bark Bakhshali Manuscript, discovered in 1881 in the village of Bakhshali, near Peshawar (modern day Pakistan) and is likely from the 7th century CE.

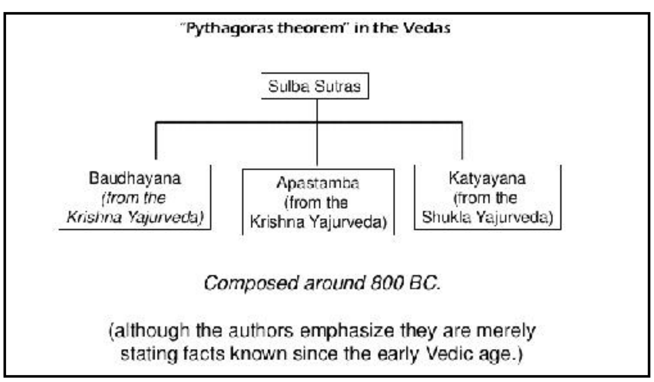


Fig. 1 : The case of the "Pythagoras theorem"

Ancient Indian was aware of the concept of zero, actually they are solely responsible for hugely important invention in mathematics and it would be no hyperbole to say that actually they invented the zero.

The earliest recorded date an inscription of zero on Sankheda Copper Plate was found in Gujrat In 585-586CE , not only this but also use of a circle which symbolizes the number zero is found in a 9th Century engraving in a temple in Gwalior in central India. But the brilliant conceptual leap to include zero as a number in its own right (not merely as a placeholder, a blank or empty space within a number, as it had been treated until that time) is usually associated to the 7th Century Indian mathematicians Brahmagupta - or possibly another Indian, Bhaskara I - even though it may well have been in practical use for centuries before that. The use of zero as a number which could be used in calculations and mathematical investigations would revolutionize mathematics.

Had there been no zero there would have been no binary number system and as a result no computers and how cumbersome would have been counting?

The Hindu culture has positional number system in base ten they used a dot to represent an empty place, sunya which meant empty was the name for this dot at this point earlier zero was a place holder and an aid in computation, by 500 C.E. the Hindus use a small circle to represent zero this circle was later on recognized as a numeral zero.

The ancient Hindu symbol of a circle with a dot in the middle, known as *bindu* or *bindhu*, symbolizing the void and the negation of the self, was probably instrumental in the use of a circle as a representation of the concept of zero.

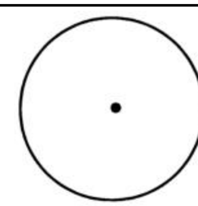


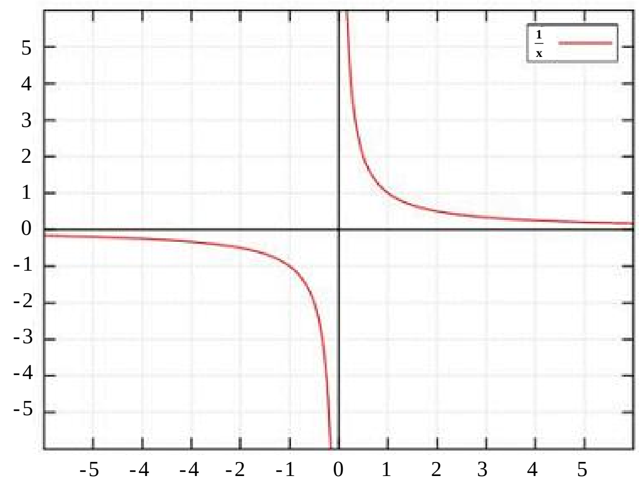
Fig. 2 : Bindu

Zero in India

The number 0 appear in the late 10th century in India. The concept of zero as a number and not merely a symbol for separation is attributed to India, where by the 9th century CE, practical calculation were carried out using zero, which was treated like any other number, even in case of division. The word for zero in Hindu-India is 'Shunya" meaning void or empty. India is recognized with great respect for its invention of Zero by importance with technological world. These are the numbers that the Hindus and the Arabics use.

Fig. 3 : Ancient Indian text explains concept of Infinity, as reciprocal of zero as well as they were also aware of the concept of trigonometric functions

As x gets bigger and bigger $\frac{1}{x}$ approaches zero. Conversely, as x gets smaller and smaller and approaches zero, $\frac{1}{x}$ approaches infinity



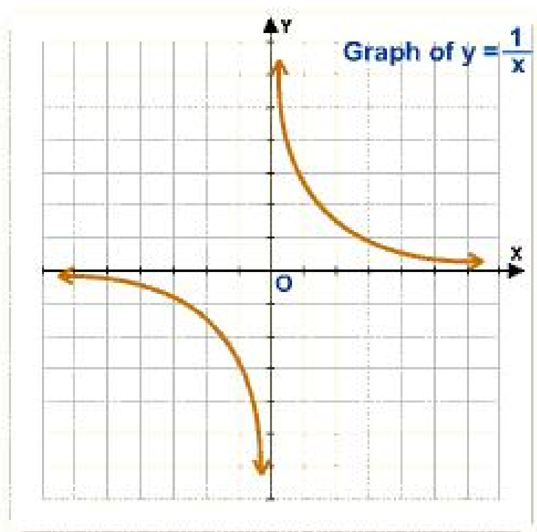


Fig. 4 : Sine approximation formula

In mathematics, Bhaskara I's sine approximation formula is a rational expression in one variable for the computation of the approximate values of the trigonometric sines discovered byBhaskara I (c. 600 – c. 680), a seventh-century Indian mathematician. This formula is given in his treatise titled *Mahabhaskariya*. It is not known how Bhaskara I arrived at his approximation formula. However, several historians of mathematics have put forward different theories as to the method Bhaskara might have used to arrive at his formula. The formula is elegant, simple and enables one to compute reasonably accurate values of trigonometric sines without using any geometry whatsoever.

The approximation formula :

The formula is given in verses 17 – 19, Chapter VII, Mahabhaskariya of Bhaskara I. A translation of the verses is given below:

(Now) I briefly state the rule (for finding the *bhujaphala* and the*kotiphala*, etc.) without making use of the Rsine-differences 225, etc. Subtract the degrees of a *bhuja* (or*koti*) from the degrees of a half circle (that is, 180 degrees). Then multiply the remainder by the degrees of the *bhuja* or*koti* and put down the result at two places. At one place subtract the result from 40500. By one-fourth of the remainder (thus, obtained), divide the result at the other place as multiplied by the '*anthyaphala* (that is, the epicyclic radius). Thus, is obtained the entire*bahuphala* (or,*kotiphala*) for the sun, moon or the star-planets. So also are obtained the direct and inverse Rsines.

(The reference "Rsine-differences 225" is an allusion to Aryabhata's sine table.)

In modern mathematical notations, for an angle x in degrees, this formula gives :

$$\sin x^0 = \frac{4x\,(180 - x)}{40500 - x\,(180 - x)}$$

Equivalent forms of the formula :

Bhaskara I's sine approximation formula can be expressed using the radian measure of angles as follows (O'Connor and Robertson, 2000) :

$$\sin x \approx \frac{16x(\pi - x)}{5\pi^2 - 4x(\pi - x)}$$

For a positive integer n this takes the following form (George, 2009) :

$$\sin \frac{\pi}{n} \approx \frac{16(n-1)}{5n^2 - 4n + 4}$$

Equivalent forms of Bhaskara I's formula have been given by almost all subsequent astronomers and mathematicians of India. For example, Brahmagupta's (598 – 668 CE) *Brhma-Sphuta-Siddhanta* (verses 23 – 24, Chapter XIV) (Gupta, 1967) gives the formula in the following form:

$$R\ \sin x^0 = \frac{Rx\,(180 - x)}{10125 - \frac{1}{4}x\,(180 - x)}$$

Also, Bhaskara II (1114 – 1185 CE) has given this formula in his Lilavati (Kshetra-vyavahara, Soka No.48) in the following form:

$$2R \sin x^0 = \frac{4 \times 2R \times 2Rx \times (360R - 2Rx)}{\frac{1}{4} \times 5 \times (360\,R)^2 - 2Rx \times (360R - 2Rx)}$$

Accuracy of the formula :

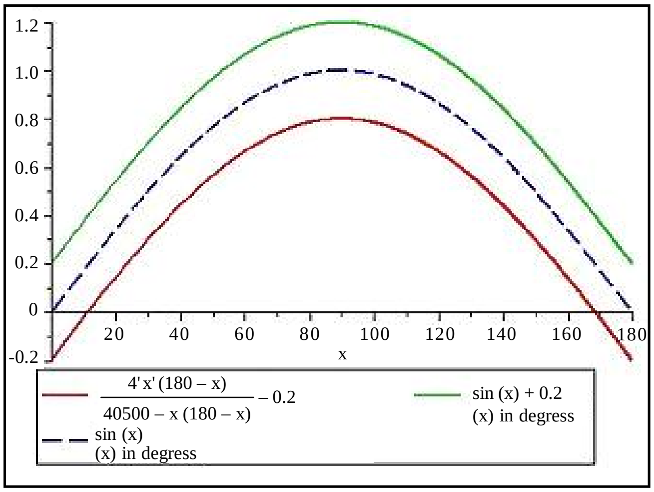


Fig. 5 : Figure illustrates the level of accuracy of the Bhaskara I's sine approximation formula. The shifted curves 4 x (180 - x) / (40500 - x (180 - x) - 0.2 and sin (x) + 0.2 look like exact copies of the curve sin (x).

The formula is applicable for values of $x°$ in the range from 0 to 180. The formula is remarkably accurate in this range. The graphs of sin (x) and the approximation formula are indistinguishable and are nearly identical. One of the accompanying figures gives the graph of the error function, namely the function :

$$\sin x^0 = \frac{4x\,(180 - x)}{40500 - x\,(180 - x)}$$

In using the formula. It shows that the maximum absolute error in using the formula is around 0.0016. From a plot of the percentage value of the absolute error, it is clear that the maximum percentage error is less than 1.8. The approximation formula thus, gives sufficiently accurate values of sines for all practical purposes. However, it was not sufficient for the more accurate computational requirements of astronomy. The search for more accurate formulas by Indian astronomers eventually led to the discovery the power series expansions of sin *x* and cos*x* by Madhava of Sangamagrama (c. 1350 – c. 1425), the founder of the Kerala school of astronomy and mathematics.

Indian astronomers have used the concept of trigonometry and its table to calculate the distance of sun from earth they defined trigonometric functions as below

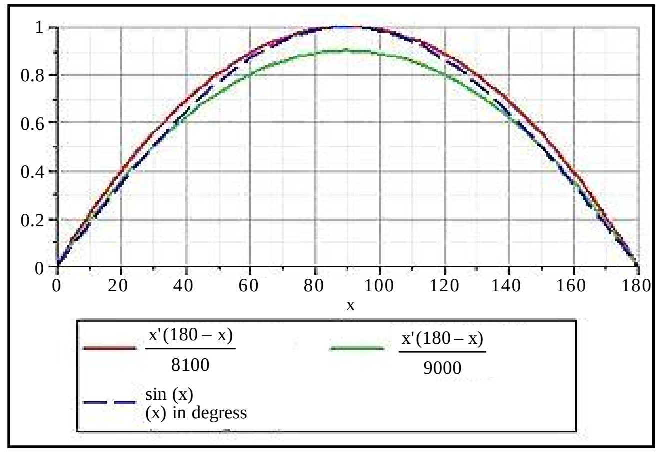


Fig. 6 : Power series

Table 1 : Trigonometric functions

Function (abbreviation)	Definition
sine (sin)	$\frac{\text{opposite}}{\text{hypotenuse}}\ \sin A = \frac{a}{c}$
cosine (cos)	$\frac{\text{adjacent}}{\text{hypotenuse}}\ \cos A = \frac{b}{c}$
tangent (tan)	$\frac{\text{opposite}}{\text{adjacent}}\ \tan A = \frac{a}{b}$
cotangent (cot or ctn)	$\frac{\text{adjacent}}{\text{opposite}}\ \cot A = \frac{b}{a}$
secant (sec)	$\frac{\text{hypotenuse}}{\text{adjacent}}\ \sec A = \frac{c}{b}$
cosecant (csc)	$\frac{\text{hypotenuse}}{\text{opposite}}\ \csc A = \frac{c}{a}$

(Table 1).

Golden Age Indian mathematicians made fundamental advances in the theory of trigonometry. They used ideas like the sine, cosine and tangent functions (which relate the angles of a triangle to the relative lengths of its sides) to survey the land around them, navigate the seas and even chart the heavens. For instance, Indian astronomers used trigonometry to calculate the relative distances between the Earth and the Moon and the Earth and the Sun. They realized that, when the Moon is half full and directly opposite the Sun, then the Sun, Moon and Earth form a right angled triangle, and were able to accurately measure the angle as $^1/_7°$. Their sine tables gave a ratio for the sides of such a triangle as 400:1, indicating that the Sun is 400 times

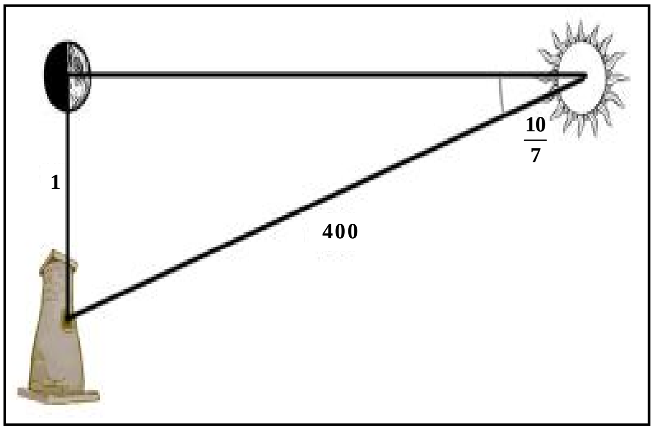


Fig. 7 : Indian trigonometry tables showed that an angle of 1/7⁰ indicate triangle sides with a ratio of 400:1, meaning that the sun is 400 times further away from the earth than the moon

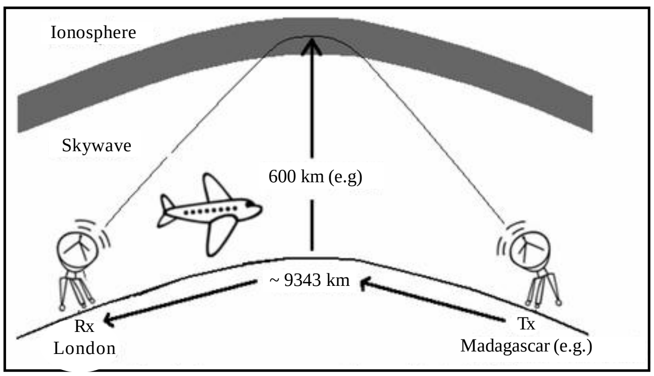


Fig. 8 : Length of skywave propagation path is 9419 km Real transmission latency is just 31.4 msec (9.4e6 / 300e6) yet a fake latency of ~250 msec is added to "satellite" comms. who decided that and why?

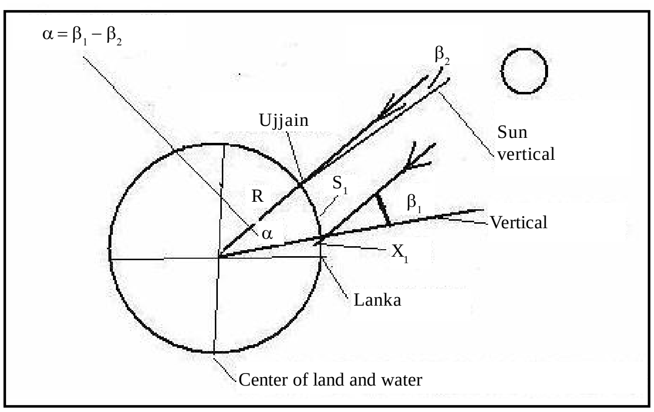


Fig. 9 : Angles made at noon at Ujjain and X1

further away from the Earth than the Moon.

Some mathematician of ancient India :

Some mathematicians of ancient India and their

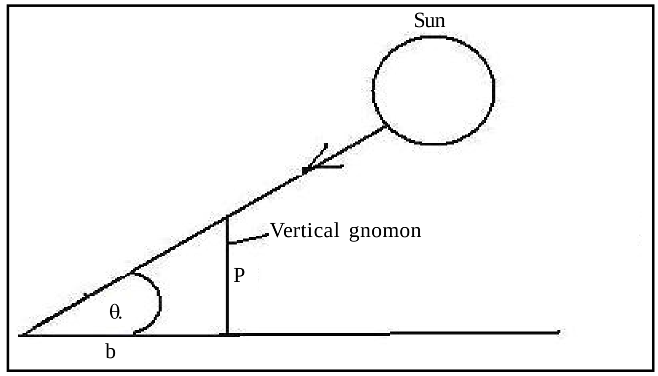


Fig. 10 : Determination of the latitude of a place on equinox day

work.

The accuracy with which Aryabhata calculated some astronomical results.

BHASKARACHARYA II (1114-1183 CE)
Genius in Algebra and Astronomy,
Contributor to world math

"In many ways, Bhasfkara represents the peak of mathematical and astronomical knowledge in the twelth centry. He reached an understanding of calculus, astronomy, the number systems, and solving equations, which were not to be achieved anywhere else in the world for several centuries."

He wrote "Bijaganita", a treatise on algebra, in which he derived a cyclic, 'Cakraval' method for solving equations of the form $ax^2 + bx + c = y$, which is usually attributed to William Brouncker (1657).

His book "lilavati" covers many branches of
mathematics, arithmetic, algebra, geometry, trigonometry and mensuration, and his treatise "Siddhant Shiromani" on astronomy contains several results in trigonometry and integral and differential calculus.

See http://www-groups.dcs.st-and.ac.uk/-history/Projects/Pearce/Chapters/Ch8 5.html
http://www.britannica.com/EBchecked/topic/64067/Bhaskara-II
http://www.newworldencyclopedia.org/entry/Bh%C4%81skara II

Fig. 11 : Bhaskaracharya II

Bhaskara II

Some of Bhaskara II's contributions are :

– A proof of the Pythagorean theorem by calculating the same area in two different ways and then canceling out terms to get $a^2 + b^2 = c^2$.
– Solution of quadratic, cubic and quartic indeterminate equation are explained.
– Solutions of indeterminate quadratic equations (of the type $ax^2 + b = y^2$).
– Integer solutions of linear and quadratic indeterminate equations.
– Solved quadratic equations with more than one unknown, and found negative and irrational solutions.
– In Siddhanta Shiromani, Bhaskara developed spherical trigonometry along with a number of other trigonometric results.

Fig. 12 : Bhaskara II

Brahmagupta gave the first rules for dealing with zero as a number

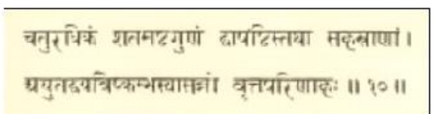
When zero is added to a number or subtracted from a number, the number remains unchanged.
A number multiplied by zero becomes zero.

... and for dealing with positive (fortune) and negative (debt) numbers

A dept minus zero is a debt.
A fortune minus zero is a fortune.
Zero minus zero is a zero.
A debt subtracted from zero is a fortune.
A fortune subtracted from zero is a debt.
The product of zero multiplied by a debt or fortune is zero.
The product of zero multiplied by zero is zero.
The product or quotient of two fortunes is one fortune.
The product or quotient of two debts is one fortune.
The product or quotient of a debt and a fortune is a debt.
The product or quotient of a fortune and a debt is a debt.

Fig. 13 : First rules for dealing with zero as a number

Aryabhata (499 A.D.)



irrationality!

- "Add 4 to 100, multiply by eight, and then add 62,000. By this rule the circumference of a circle with a diameter of 20,000 *can be approached.*"
- The Aryabhata algorithm, further developed later by Bhaskara in 621, is the standard method for solving first order Diaphontine equations (ax+by=c). Also known as Kuttaka (pulverizer) algorithm.
- Modern methods for finding square roots and cube roots, these methods were introduced in Europe only in the 16th century AD. (*Amartya Kumar Datta, Resonance, April 2002.*)

Fig. 14 : Aryabhata (499 A.D.)

Aryabhata wrote many mathematical and astronomical treatises. His chief work was the

Astronomy : Aryabhata

Period of rotation around the Sun

Planet	Aryabhata's Value	Modern
Earth, Year	365.259days	365.256days
Moon,	27.322days	27.322days
Mars,	1.881 yrs	1.881 yrs
Jupiter,	11.861 yrs	11.862 yrs
Saturn	29.477 yrs	29.458 yrs

Fig. 15 : Astronomy : Aryabhata

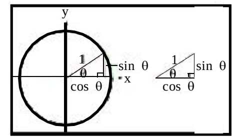
'Ayrabhatiya' which was a compilation of mathematics and astronomy. The name of this treatise was not given to it by Aryabhata but by later commentators. A disciple by him called the 'Bhaskara' names it 'Ashmakatanra' meaning 'treatise from the Ashmaka'. This treatise is also referred to as 'Ayra-shatas-ashta' which translates to 'Aryabhata's 108'. This is a very literal name because the treatise did in fact consist of 108 verses. It covers several branches of mathematics such as algebra, arithmetic, plane and spherical trigonometry. Also included in it are theories on continued fractions, sum of power series, sine tables and quadratic equations.

Aryabhata worked on the place value system using letters to signify numbers and stating qualities. He also came up with an approximation of pi ( ) and area of a triangle. He introduced the concept of sine in his work called 'Ardha-jya' which is translated as 'half-chord'.

One of the most significant input of Brahmagupta to mathematics was the introduction of 'zero' to the number system which stood for 'nothing'. His work the 'Brahmasphutasiddhanta' contained many mathematical findings written in verse form. It had many rules of

Bhaskara I (600-680 AD)

- Important works – MahAbhAskarIya and LaghubhAskarIya (provided explanations and interpretations of Aryabhata's reasonings).
- AryabhatIyabhAshya – a commentary on Aryabhatia (dated 628 AD.).
- Provided a compact classification of mathematics into different specializations (Encyclopedia).



- Responsible for evolving trigonometry in its present form (*ardhajya* etc. see encyclopedia), and created the modern trigonometric circle.
- Gave an approximation for the sine. $\boxed{\sin x \approx \dfrac{16x(\pi - x)}{5\pi^2 - 4x(\pi - x)'}(0 \le x \le \dfrac{\pi}{2})}$
- Elaborated on the *kuttaka* method of Aryabhata.

Fig. 16 : Bhaskara I (600-680 AD)

Brahmagupta:

- Brahmagupta was one of the great Indian mathematician and astronomer who wrote many important works on mathematics and astronomy.
- His best known work is the *Brahmasphutasiddhanta* written in AD 628.
- Brahmagupta was the first to use zero as a number. He gave rules to compute with zero.
- Brahmagupta used negative numbers and zero for computing. The modern rule that two negative numbers multiplied together equals a positive number first appears in *Brahmasphutasiddhanta*.

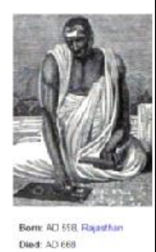


Fig. 16 : Brahmagupta

arithmetic which is part of the mathematical solutions now. These are 'A positive number multiplied by a positive number is positive.', 'A positive number multiplied by a negative number is negative', 'A negative number multiplied by a positive number is negative' and 'A negative number multiplied by a negative number is positive'. The book also consisted of many geometrical theories like the 'Pythagorean Theorem' for a right angle triangle. Brahmagupta was the one to give the area of a triangle and the important rules of trigonometry such as values of the sin function. He introduced the formula for cyclic quadrilaterals. He also gave the value of 'Pi' as square root ten to be accurate and 3 as the practical value. Additionally he introduced the concept of negative numbers.

Brahmagupta solved the so-called Pell equation $nx^2 + 1 = y^2$.

In modern notation, the solution he gave was

$$x = \frac{2t}{t^2 - n} \text{ and } y = \frac{t^2 + n}{t^2 - n}$$

where, t could be replaced by any number.

For example, if t = 3, the we get the solution

$$x = \frac{6}{9 - n} \text{ and } y = \frac{9 + n}{9 - n}$$, so that :

$$nx^2 + 1 = n\left(\frac{6}{9 - n}\right)^2 + 1 = \frac{36n}{81 - 18n + n^2}$$

$$+ 1 = \frac{36n + 81 - 18n + n^2}{81 - 18n + n^2} = \frac{81 + 18n + n^2}{81 - 18n + n^2} = \left(\frac{9 + n}{9 - n}\right)^2 = y^2$$

The Surya Siddhanta , a textbook on astronomy of ancient India which was last complied in 1000B.C. and is believed to be handed down from 3000 B.C. by aid of complex mnemonic recital method still known today , it clearly shows the Earth's Diameter to be 7,840 miles as compared to modern method which found it be 7926,7 miles , it also shows distance between Earth and Moon to be 253,000 miles whereas the same distance as calculated by modern methods comes out too be 252,710 miles .

It clearly shows the high accuracy acquired by the mathematician of ancient India

Conclusion :

I wish to conclude firstly by simply saying that the work of Indian mathematicians has been severely neglected by our own historians (so called secular and left oriented) and in whose view neglecting and abusing our great heritage was only basis for their recognition and western historians as well, although the situation is improving a little bit now. Two questions, firstly, why have Indian works been neglected and the secondary question, shouldn't this neglect be considered a great injustice? These has to be answered then and only then we will be able appreciate and revive what had been done by our mathematicians of Ancient India.

What I primarily wished to tackle was to answer that is, what appears to have been the motivations and aims of scholars who have contributed to the Eurocentric view of mathematical history. This leads to the way ancient Indian Mathematics has been neglected and ignored by Eurocentric scholars and scientist who thought Indians pagans religious and Mathematics which is mentioned in their texts etc could not be the basis of their monotheistic culture, and these scholars believed that this is true ever since, but the modern researches and evidences are proving that vast majority of works has been done by Indians in ancient age and most of the books are written in Sanskrit or Hindi.

In last I only want to say that there is a vast scope for study of *VEDIC MATHEMATICS* and it should be definitely studied only because there are much evidences that Indian mathematics has played a very important role in the history of mathematics, much more work should be done to analyze ancient documents and archeological evidences of a very intelligent society.

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