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RESEARCH PAPER

Domain studies with imperfect frame in large population

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Abstract : Existence of the frame is pre-requisite for any sample survey or census of a large population. Frames are quite often imperfect due to dynamic nature of sampling units. Frames become incomplete by the time actual survey and enumeration starts which affects the statistical results desired for the target population. In present study imperfection in the frame of large population arising due to qualitative change of units from one class to other have been considered. We have considered incomplete frame assuming the nature of units following dynamic change from class one to other follow a probability distribution function. Suitable estimator for proportion of units belonging to a particular domain and unbiased estimate of target population for a class have been proposed along with its estimate of variance. The estimates are evolved so as to eliminate error caused due to deviation of sampled population from target population. The paper deals with interesting problem arising in survey sampling and is useful in practice.

Key Words : Imperfect frame, Sampled Population, Target population, Out-dated units

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INTRODUCTION

The purpose of the statistical survey is to obtain information about population by selecting units from the frame available and prepared at some time. The samples are expected to be representative of the population. The population is aggregate of all sampling units determined and defined according to the aim of the survey depending upon some rule. Sampling units must be distinct, identifiable, non-over lapping, unambiguous and must cover entire population. The probability selection procedure is directly applied to such sampling units according to the purpose of the statistical analysis. We first, determine the population on which statistics is to supply information. Thus, population under study is called target population. For practical analysis of the target population in complete census as well as in sample survey, some means are needed to permit one to attain the reporting units of this population which either exist as a units or which have to be created for the survey. The aggregate of sampling units defines a population, which in order to distinguish it from the target population, will be referred as sampled population.

The results of the statistical analysis are always valid only for sampled population which correspond to the list of sampling units, no matter whether conclusions on the sampled population are based on the sample survey or complete causes. The purpose of the statistics is however, to provide exact information on the target population. Therefore, the population to be sampled (the sampled population) should coincide with the population about which information is wanted (the target population). To what extent, the conclusions drawn from the sampled population will also apply to the target population must depend upon the other sources of information. Hence, supplementary information about the nature of difference between sampled and target population must be gathered. Such information is available on the structure of the frame which is list of all the sampling units. Therefore, our first requisite is the existence of complete frame with reference to which relevant data are to be collected and represented consisting of description of all sampling units. The nature and details of the frame become the basis for the choice of appropriate sampling design. But frame, so constructed are often found to be incomplete, out-dated, illegible and contains unknown duplication. Bitter experience show that sampler have acquired a critical attitude towards the list that have been routinely collected for some purpose. Thus, there are rare situations in practice where frame in available in the form sampler desires to use. Incompleteness is common to virtually all lists used for sampling mainly due to dynamic nature of the population.

Therefore, conclusions based on the sampled population which corresponds to the frame actually applied in census or survey amounts error and bias for the results of the target population, because of imperfection of the frame.

The precision of the statistical results has to be judged by total error, which as to the results to be obtained for target population consists of following components.

(i) Error due to deviation of target population from sampled population due to imperfection of the frame (ii) Sampling error and (iii) Non Sampling error. Errors caused due to component (ii) and (iii) have so far been largely discussed and analyzed in the literature. But error caused due to component (i) have exceptionally been discussed and no much work have been done in this direction. It is in the light of component (i) that draws attention of present study rarely discussed so far.

Error (i) arises from the fact that the sampled population to which results refer and the target population for which results are needed do not conform to each other due to imperfection and incompleteness of the frame. This error can broadly be classified as deviation of coverage and deviation of content. Error of deviation of coverage may occur due to omission of sampling units from the target population, sampling units being out of scope or out dated due to dynamic nature of the frame and duplication of units. Error due to deviation in content may occur when available frame provides incorrect auxiliary information on reporting units. By the time of actual survey, there is seldom one to one correspondence between units of sampled population and target population because of the dynamic nature of the population. Selection of sample from such imperfect frame will not subscribe to the principle of random selection as some of the out dated units which should have been assigned zero probability of selection have also been selected in the sample and it was discovered because unit was selected in the sample. Quite often a frame consists of some superfluous units which do not exist in the target population at the time of actual enumeration and for which rule of association do not lead to any of the reporting unit in the target population. The listed units which are associated with some reporting unit may be identical in all the respects so that imperfection arises due to duplication of units in the frame. Error caused due to imperfection of frame is serious when exceedingly large unit is incorrectly assigned a small measure of size when selection is with probability proportional to measure of size and error is discovered because the unit was selected.

Thus, imperfect frames are the rule rather than the exceptions. However, sampling theory appears to have been evolved largely around perfect frame based on ideal conditions. But in practice, the frame available are often incomplete and imperfect and will not conform in its delimitations and compositions to the target population. It remains, never the less, the aim of the statistics to supply information as exact as possible on the target population.

Seal (1962) discussed the use of out dated frames in large scale surveys and considered the changes in the population as a continuous stochastic process. Hartely (1962) proposed the use of two or more frames to overcome the problem of incomplete frames. Hansen, Hurwitz and Jabine (1963) discussed various procedures for the use of incomplete frame and proposed the predecessor-successor method to obtain information on missing units in the frame. Szamsitat and Schaffer (1963) discussed about consequences of imperfect frame in sampling. Singh (1983) gave a mathematical formulation of predecessor-successor method for estimating total number of units missing from the frame. Singh *et al.* (1997) discussed imperfection in the frame of finite population and proposed estimators for domain of study considering probability distribution of the out-dated units in the incomplete frame. Singh (2020) discussed the frame error as error due to imperfection of the frame in detail because of deviation of the target population from sample population.

In many situations, we not only need estimate of the population of large size but also want separate estimates for different segment of the population. Therefore, it is desirable to have separate frame for different segment. But at the time of actual survey, there may be qualitative change of the units so that frames of different segment becomes out-dated and imperfect. Some times, we may be interested in particular class of the population of large size so that we need frame of that class and use the same for selecting the sample. But due to qualitative change of the units, the frame of a class may be imperfect as units may change from one class to other rapidly and is discovered only when a unit is selected and investigated. For example, some unirrigated area may be found irrigated and some irrigated area as listed in the frame may be discovered as unirrigated at the enumeration stage, seasonal and landless laborers may migrate from one place to other and during passage of time in the large population and *vice-versa.* Nomadic tribes listed in the frame may be non-existent in the target population of a locality and some new nomadic tribe from other class may migrate to that class. Frame of persons susceptible to H.I.V. may be out dated at the time of actual study as some persons from the frame of non-susceptible to H.I.V. may be discovered as susceptible to H.I.V. and *vice versa*. In a demographic study of population control programme, the frame of persons following family planning measures may contain some out-dated unit in the frame so contracted earlier at the time of actual enumerations as some persons may be out of reproductive period or may be discovered as not abiding with such measures and *vice-versa.* In the poverty alleviation programme, the frame of persons below poverty line may be imperfect and out dated as some persons above poverty line as listed may now be discovered as below poverty line and vice-versa. Such units are discovered as out dated united because they were selected in the sample and enumerated. It is also obvious because frame, even if prepared with paramount precaution, is time taking and by the time actual survey starts, the frame becomes out dated and imperfect.

Present paper deals with such interesting problem arising in the survey sampling of a large population which is useful in practice.

Methods of estimation :

Consider a large population of size N and let N_1 units of the sampled population belong to class I and N- N_1=N_2 belong to other class or remaining classes. Assume the separate frame of and are available so that units are identifiable, mutually exclusive and non-overlapping. However, during passage of time and due to dynamic nature of units, there is qualitative change of units from one class to other such that population consists of units belonging to class I and units belonging to class II. Sampler does not have prior information about sizes of and at the time of sample selection. In very large variety of investigations, sample selected of some size is such that population size N, and may be regarded as infinite. Assume that. Thus, the frame available of units may contain some units which actually do not belong to the target population of class I and also some of the units belonging to class I may not be listed in the frame of that class at the time of actual survey.

Let denote the number of units of the target population which actually belong to the class I, out of belonging to class I of the old available frame. Let be the number of units which have gone qualitative change from class I to class II out of N_1 units so that. Similarly, denote the number of units from the samples population of size, which belong to class II and N_{21} be the number of units which have changed from class II to I in the old available frame but units are not listed in class I so that . Therefore the actual complete frame of target population consists of for class I and for class II.

We may write

$N_1' = N_{11} + N_{21}$: Units actually belonging to the target population of class I at the time of actual survey. (2.1)

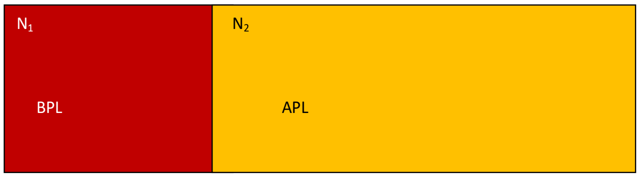
$N_2' = N_{22} + N_{12}$: Units actually belonging to class II at the time of survey in the target population.

For example:

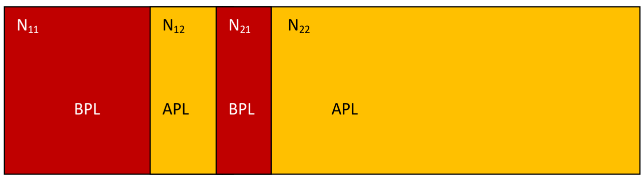
Assume that the population of size N constitutes N_1 individuals below poverty line (BPL) and N_2 individuals are above poverty line (APL) so that, N =N_1+N_2 at time t_1. During passage of time, the frame changes so that, at time t_2 BPL size is and APL size as which are unknown and N = $N_1' + N_2'$

It can be illustrated by following diagram:

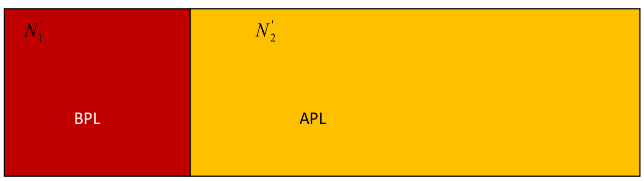
Frame at time t_1



Frame at time t_2



Target population



Further we define

Further we define $P_{11} = \frac{N_{11}}{N_1}$: Proportion of units of the target population belonging to class I out of N_1 units belonging to class I in the sampled population.

Similarly we may have

$$P_{22} = \frac{N_{22}}{N_2},\ P_{12} = \frac{N_{12}}{N_1},\ N_{21} = \frac{N_{21}}{N_2} \qquad (2.3)$$

Since, we have information about N_1 and N_2 units available from the imperfect frame corresponding to sampled population. We have to estimate $N_1^{'}$. Therefore, N_{11} and N_{21} are to be estimated. Further let y denote the characteristic under study. We define

$Y_{N_1^{'}} = \Sigma y_i$: i=1,2,3…,: $N_1^{'}$ Population total for y for $N_1^{'}$ units belonging to class I

$\bar{Y}_{11} = \frac{1}{N_{11}} \Sigma y_i$: i=1,2,3…, N_{11}: Population mean for y for units belonging to class I

(2.3) $\bar{Y}_{21} = \frac{1}{N_{21}} \Sigma y_i$: i=1,2,3…, N_{21}: Population mean for y for N_{21} units which have changed from class II to class I. The target population total for class I may be written as :

$$Y_{N_1^{'}} = N_{11}\bar{Y}_{11} + N_{21}\bar{Y}_{22} \qquad (2.4)$$

Thus in order to obtain an estimator of the target population total for y of class I. We have to estimate N_{11}, Y_{11}, N_{21} and Y_{21}.

Proposed sampling scheme:

Select a sample of sizefrom the accessible imperfect frame of large size by SRSWOR. But as some units of the sampled population of class I have gone qualitative change in the sense that some units belong to the target population of class II, therefore, some of the units selected in the sample of size n_1 may not actually belong to class I corresponding to the target population of that class. Let n_{11} denote the number of units which belong to class I as discovered at the enumerating stage and are those units in the sample which were once in class I but now are found to be belonging to class II because of qualitative change of sampling units, unknown of course, at the time of sample selection from the imperfect frame. Now unbiased estimator of N_{11} and N_{12} can be easily given by $\hat{N}_{11} = \frac{n_{11}}{n_1 N_1}$, $\hat{N}_{12} = \frac{n_{12}}{n_1 N_1}$, where $n_{11} + n_{12} = n_1$

In order to estimate, let us select a sample of size n_2 from imperfect frame of known size N_2 with SRSWOR from class II of sampled population. The every unit of size n_2 is investigated by the enumerator. Let, it is discovered that n_{21} units have gone qualitative change from class II to class I so that they exist in target population of class I. similarly, let n_{22} are units which actually exist in class II so that $n_2 = n_{22} + n_{21}$. However, if information about out-dated units were available at the time of sample selection from old frame, such out dated units could have been assigned zero probability of selection and deleted from the sample. But, most often we do not know about such dynamic units unless actual enumeration starts and it is only when enumerator visits particular unit that he finds that unit no more exist in the desired target population (domain of study). Therefore, we have proposed an alternative method for selecting the unit and estimation procedure. Thus, we can have unbiased estimators of N_{21} and N_{22} as :

$$\hat{N}_{21} = \frac{n_{21}}{n_2 N_2} \text{ and } \hat{N}_{22} = \frac{n_{22}}{n_2 N_2} \text{ where } n_{21} + n_{22} = n_2 \qquad (3.2)$$

Now for the large population, the probability $P(n_{11})$ that in a sample of size n_1 selected from large population of size N_1, n_{11} will belong to class I and in class II is given by the Binomial distribution.

Therefore we have

$$P(n_{11}) = \binom{n_1}{n_{11}} p_{11}^{n_{11}} (1-p_{11})^{n_{12}} \quad n_{11} = 0,1,2,....,n_1$$

Thus we have

$$E(n_{11}) = n_1 p_{11}, \; V(n_{11}) = n_1 p_{11} q_{11}, \; q_{11} = 1 - p_{11}$$

$$E(n_{11}^2) = n_1(n_1 - 1)p_{11}^2 + n_1 p_{11}$$

$$p(n_{21}) = \binom{n_1}{n_{11}} p_{21}^{n_{21}} (1-p_{21})^{n_{22}}, \quad n_{21} = 0,1,2,....,n_2$$

$$E(n_{21}) = n_2 p_{21}, \; V(n_{21}) = n_2 p_{21} q_{21}, \; q_{21} = 1 - p_{21}$$

$$E(n_{21}^2) = n_2(n_2 - 1)p_{21}^2 + n_2 p_{21}$$

Therefore, $\hat{N}_{11}, \hat{N}_{12}, \hat{N}_{22}$ and $\hat{N}_{21}$ are unbiased estimates. The unbiased estimator of $Y_{N_1'}$ can be given by $Y_{N_1'} = (\frac{n_{11}}{n_1})N_1 \bar{y}_{11} + (\frac{n_{21}}{n_2})N_2 \bar{y}_{21}$

where,

$\bar{y}_{11} = \frac{1}{n_{11}} \Sigma y_i$: $(i = 1,2,3,....,n_{11})$ and $\bar{y}_{21} = \frac{1}{n_{21}} \Sigma y_i$: $(i = 1,2,3,....,n_{21})$ are unbiased estimators of $\bar{y}_{11}$ and $\bar{y}_{21}$, respectively.

It can be easily seen that $E(\hat{y}_{N_1'}) = Y_{N_1'}$

Variance of $V(Y_{N_1'})$

The variance of the estimate of target population total can be given as:

$$V(Y_{N_1'}) = V\{(\frac{n_{11}}{n_1})N_1 \bar{y}_{11}\} + V\{(\frac{n_{21}}{n_1})N_2 \bar{y}_{21}\} \quad (4.1)$$

Since $\bar{y}_{11}, \bar{y}_{21}, n_{11}$ and n_{21} are independent, we may also write

$$V(Y_{N_1'}) = \frac{N_1^2}{n_1^2} V(n_{11}\bar{y}_{11}) + \frac{N_2^2}{n_2^2} V(n_{21}\bar{y}_{21}) \quad (4.2)$$

Now $V(n_{11}\bar{y}_{11}) = E\{V(n_{11}\bar{y}_{11} \mid n_{11})\} + V\{E(n_{11}\bar{y}_{11} \mid n_{11})\}$

$$= E\{\frac{N_{11}^2 (N_{11} - n_{11}) S_{11}^2}{N_{11} n_{11}}\} + \bar{Y}_{11}^2 V(n_{11})\}$$

$$= E[\{\frac{(N_{11} n_{11} - n_{11}^2)}{N_{11}}\} S_{11}^2] + \bar{Y}_{11}^2 n_1 p_{11} q_{11}$$

$$= (\frac{n_1 p_{11} S_{11}^2}{N_{11}})(N_{11} - n_1 p_{11} + p_{11} - 1) + n_2 p_{11} q_{11} \bar{y}_{11}^2$$

where,

$$S_{11}^2 = \{\frac{1}{N_{11} - 1}\} \Sigma (y_i - \bar{Y}_{11})^2 : i = 1,2,3,...., N_{11}$$

Therefore, by putting $N_{11} = N_1 p_{11}$, we have after simplification

$$(\frac{N_1^2}{n_1^2}) V(n_{11} y_{11}) = \frac{N_1}{n_1} [S_{11}^2 \{p_{11}(N_1 - n_1 + 1) - 1\} + N_1 p_{11}(1 - p_{11}) \bar{Y}_{11}^2)] \quad (4.3)$$

Similarly we can have

$$(\frac{N_2^2}{n_2^2}) V(n_{21} y_{21}) = \frac{N_2}{n_{21}} [S_{21}^2 \{p_{21}(N_2 - n_2 + 1) - 1\} + N_2 p_{21}(1 - p_{21}) \bar{Y}_{21}^2)] \quad (4.4)$$

Therefore, from (4.3) and (4.4) we obtain

$$V(\hat{Y}_{N_1'}) = \frac{N_1}{n_1} S_{11}^2 \{p_{11}(N_1 - n_1 + 1) - 1\} + N_1^2 p_{11}(1 - p_{11}) \frac{\bar{Y}_{11}^2}{n_1} + \frac{N_2}{n_2} S_{21}^2 \{p_{21}(N_2 - n_2 + 1) - 1\} + N_2^2 p_{21}(1 - p_{21}) \frac{\bar{Y}_{21}^2}{n_2} \quad (4.5)$$

Thus, variance of $\hat{Y}_{N_1'}$ depends on $N_1, n_1, p_{11}, S_{11}, \bar{Y}_{11}$ and $N_2, n_2, p_{21}, S_{21}, \bar{Y}_{21}$ but not on the $S_1, N_1, n_1, S_2 N_2, n_2$ which is normally believed and followed. However, if $p_{11} = 1$ and $p_{21} = 1$ so that there is no imperfection in the frame, then we have $V(\hat{Y}_{N_1}') = \frac{N_1(N_1 - n_1)S_1^2}{n_1}$ as expected since for $p_{11} = 1$ $S_{11}^2 = S_1^2$ and for $p_{21} = 0$ and $S_{21} = 0$.

Therefore, $v(\hat{Y}_{N_1}')$ will depend on the nature of p_{11} and p_{21}. The given formula is free from error caused due to deviation of sampled population from target population. This work can be extended for other domain studies.

Estimation of $V(\hat{Y}_{N_1'})$

Estimate of can be obtained by estimating each R.H.S. terms in the eq. (4.5).

Therefore we have

$$V(\hat{Y}_{N_1'}) = \frac{N_1}{n_1} Est.S_{11}^2 \{Est.p_{11}(N_1 - n_1 + 1) - 1\} + \frac{N_1^2}{n_1} Est.p_{11}(1 - p_{11}) \, Est.\bar{Y}_{11}^2 + \frac{N_2}{n_2} Est.S_{21}^2 \{Est.p_{21}(N_2 - n_2 + 1) - 1\} + \frac{N_2^2}{n_2} Est.p_{21}(1 - p_{21}) \, Est.\bar{Y}_{21}^2 \quad (5.1)$$

Now we know that

$Est.S_{11}^2 = s_{11}^2$, $Est.p_{11} = \hat{p}_{11} = \frac{n_{11}}{n_1}$, as $E(\frac{n_{11}}{n_1}) = p_{11}$

$Est.S_{21}^2 = s_{21}^2$, $Est.p_{21} = \hat{p}_{21} = \frac{n_{21}}{n_2}$, as $E(\frac{n_{21}}{n_2}) = p_{21}$

Again we have

$$\text{Est.}\overline{Y}_{11}^2 = \overline{y}_{11}^2 - \{\frac{N_{11} - n_{11}}{N_{11}n_{11}}\}s_{11}^2 \text{ and } s_{11}^2 = \frac{1}{n_{11}}\Sigma(y_1 - \overline{y}_{11})^2$$

$$\text{Est.}\overline{Y}_{21}^2 = \overline{y}_{21}^2 - \{\frac{N_{21} - n_{21}}{N_{21}n_{21}}\}s_{21}^2 \text{ and } s_{21}^2 = \frac{1}{n_{21}}\Sigma(y_1 - \overline{y}_{21})^2$$

Again

$$\text{Est.}p_{11}(1 - p_{11}) = \{\frac{n_1}{n_1 - 1}\}\{\hat{p}_{11}(1 - \hat{p}_{11})$$

Because

$$E\{\frac{n_1}{n_1 - 1}\hat{p}_{11}(1 - \hat{p}_{11})\} = \{\frac{n_1}{n_1 - 1}\}\{E(\hat{p}_{11}) - E(\hat{p}_{11}^2)\}$$

$$= \frac{n_1}{n_1 - 1}[p_{11} - \{n_1(n_1 - 1)p_{11}^2 + n_1 p_{11}\}/n_1^2)]$$

$$= p_{11}(1 - p_{11}$$

Similarly

$$Est.p_{21}(1 - p_{21}) = \frac{n_2}{n_2 - 1}[\hat{p}_{21} - (1 - p_{21}^2)]$$

Putting these values in (5.1) we obtain

$$\hat{V}(\hat{Y}_{N_1'}) = N_1 s_{11}^2[\{(N_1 - n_1 + 1)\hat{p}_{11} - 1\}/n_1 - \{N_1\hat{p}_{11}(1 - \hat{p}_{11})(N_{11} - n_{11})\}$$
$$/(n_1 - 1)N_{11}n_{11}] + \{N_1^2\hat{p}_{11}(1 - \hat{p}_{11})\overline{y}_{11}^2\}/(n_1 - 1)$$
$$+ N_2 s_{21}^2[\{(N_2 - n_2 + 1)\hat{p}_{21} - 1\}/n_2 - \{N_2\hat{p}_{21}(1 - \hat{p}_{21})(N_{21} - n_{21})\}$$
$$/(n_2 - 1)N_{21}n_{21}] + \{N_2^2\hat{p}_{21}(1 - \hat{p}_{21})\overline{y}_{21}^2\}/(n_2 - 1) \qquad (5.2)$$

This value depends on the nature $\hat{p}_{11}$ of and $\hat{p}_{21}$. In case of perfect frame *i.e.* when $\hat{p}_{11} = 1$ and $\hat{p}_{21} = 0$ We obtain $\hat{V}(\hat{Y}_{N_1'}) = \frac{N_1(N_1 - n_1)s_1^2}{n_1}$ as expected because, $\hat{p}_{11} = 1$, $s_{11}^2 = s_1^2$. But (5.2) involves $\frac{1}{N_{11}}$ and $\frac{1}{N_{21}}$ which are unknown in case of imperfect and dynamic frame. Although sampler have information of n_{11} and n_{21} after sample is selected and enumerated. However, values of $\frac{1}{N_{11}}$ and $\frac{1}{N_{21}}$ can be substituted with their estimates.

Assuming $\hat{N}_{11} = N_{11} + \varepsilon$ where $E(\varepsilon) = 0$

$$E(\hat{N}_{11})^{-1} = E[N_{11}^{-1}\{1 + \frac{\varepsilon}{N_{11}}\}^{-1}]$$

$= E(\frac{1}{N_{11}}) = 0$, neglecting terms higher than power one.

$$= \frac{1}{N_{11}}$$

Therefore

$$\text{Est.}\frac{1}{N_{11}} = \frac{1}{\hat{N}_{11}} = \frac{1}{N_1\hat{p}_{11}}$$

Similarly

$$\text{Est.}\frac{1}{N_{21}} = \frac{1}{\hat{N}_{21}} = \frac{1}{N_2\hat{p}_{21}}$$

Inserting these in (5.2) we obtain

$$\hat{V}(\hat{Y}_{N_1'}) = N_1 s_{11}^2[\{(N_1 - n_1 + 1)\hat{p}_{11} - 1\}/n_1 - (N_1\hat{p}_{11} - n_{11})(1 - \hat{p}_{11})/$$
$$/(n_1 - 1)n_{11}] + N_2 s_{21}^2[\{(N_2 - n_2 + 1)(\hat{p}_{21} - 1)/n_2 - (N_2\hat{p}_{21} - n_{21})$$
$$(1 - \hat{p}_{21})/(n_2 - 1)n_{21}] + N_1^2\hat{p}_{11}(1 - \hat{p}_{11})\overline{y}_{11}^2/(n_1 - 1) + N_2^2\hat{p}_{21}(1 - \hat{p}_{21})\overline{y}_{21}^2$$
$$/(n_2 - 1) \qquad (5.3)$$

Thus, estimate of $\hat{V}(\hat{Y}_{N_1'})$ depends on known values of $N_1, N_2, s_{11}, s_{21}, \hat{p}_{11}, \hat{p}_{21}, n_1, n_{11}, n_{21}, \overline{y}_{21}$ and n_{21} determined from the samples of large population with imperfect frame. The formula derived is free from error of deviation of sampled population from target population. It is interesting to note that in case of ideal frame with no imperfection, $\hat{p}_{11} = 1$ and $\hat{p}_{21} = 0$, we have $\hat{V}(\hat{Y}_{N_1'}) = N_1(N_1 - n_1)s_{11}^2/n_1$ which can be obtained in the case of perfect frame with SRSWOR such that $s_{11}^2 = s_1^2$. Similarly estimate of $\hat{V}(\hat{Y}_{N_2'})$ can be obtained. Thus, study can be extended for other domain studies which is useful in practice.

Conclusion:

If we have two classes I and II in which there is rapid qualitative change of units from one class to other, the frame are often incomplete at the time of actual survey due to dynamic nature of units. First time we have considered incomplete frame assuming the nature of units following dynamic change from class one to other follows a probability distribution function. To estimate, population total and variance, we can find unbiased estimate of the parameters by above devised methods even with incomplete frame assuming the nature of units following dynamic change from class one to other follows a probability distribution function. In present study, it is assumed that it follows binomial distribution as population size is large. This may be useful to estimate the class where most often units change their class rapidly, by the time survey actually survey starts leading to incompleteness of frame of that class.

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