ETI ENGINEERING AND TECHNOLOGY IN INDIA
Volume 3 | Issue 2 | October, 2012 | 131-135

RESEARCH ARTICLE

Mathematical modeling for cooling by water evaporation over roof of a greenhouse

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ABSTRACT : A mathematical model with moving water film over the roof of even span model greenhouse has been developed to study the effectiveness of cooling in the greenhouse. Analytical expressions for flowing water temperature in south and north roof along with greenhouse room air temperature have been derived in terms of design and climatic parameters for summer period. The analysis is based on steady state mode. Flow of thin film of water is maintained over the jute cloth stretched on the roofs of greenhouse. The effects of relative humidity, flow rate of water, absorptivity of shading material (jute cloth) and length of roof on the cooling performance of greenhouse room air temperature are discussed thoroughly with the help of this model.

KEY WORDS : Solar energy, Greenhouse, Evaporative cooling, Thermal modeling

Article Chronicle : Received : 19.05.2012; **Revised :** 28.09.2012; **Accepted :** 20.10.2012

How to cite this Article : Ghosal, M.K. and Das, R.K. (2012). Mathematical modeling for cooling by water evaporation over roof of a greenhouse. *Engg. & Tech. in India,* **3**(2) : 131-135.

INTRODUCTION

The conventional method for reduction of heat flux into the greenhouse through roof is by the use of movable canvas (shading material) (Al-Arifi *et al.*, 1999). The stretching of canvas over the roof during the daytime and removal of it in nighttime is the effective and economical technique for protecting greenhouse from peak summer period. The heat flux through the roof of any structure can also be decreased substantially if water is evaporated on the surface of the roof (Cooper, 1973) as roof surface receives maximum amount of solar radiation (about 50 per cent of the total radiation) in summer and hence, contributes the maximum of cooling load (Duffie and Backman, 1991). Thus, evaporation of water can be achieved by maintaining a thin film of water over the surface of the roof. If water temperature is below that of shading cloth, energy is transferred from the cloth to the water raising the water temperature and lowering the cloth temperature. Also if the relative humidity is below 100 per cent, evaporation of water transfers a large amount of energy (2300 KJ/kg. of water evaporated) from cloth to atmosphere resulting in increase of convective heat transfer coefficient between them. Morris *et al.* (1958) found that internal air temperature was reduced when using a water film over the glass surface of the greenhouse due to the lowering of the temperature of the glass surface. Recently Willits and Peet (2000) conducted an experiment with intermittent application of water to an externally mounted greenhouse shade cloth and found that the rise of greenhouse air temperature was reduced by 41 per cent under wet cloth as compared to 18 per cent under dry cloth. Hence considering the efficacy of evaporative cooling system in reducing the heat flux into the greenhouse through the roof, an attempt was made to develop a thermal model to simulate the performance for cooling of greenhouse with moving water film in the roof.

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EXPERIMENTAL PROCEDURE

Working principle and description of even span model greenhouse :

By keeping the exterior surface of the roof wet, the sensible heat of the roof surface gets converted into latent heat of vaporization for evaporation of water. Water-film concept can be achieved in the roof of greenhouse by using well-knitted jute cloth over which the thin film of water is allowed to flow. Owing to the porous property of jute cloth, it behaves like a free water surface for evaporation when keeping it wet. Solar radiation when falls on the free water surface gets reflected and the rest is absorbed for evaporation of water. A little part of the absorbed radiation that is transmitted to the greenhouse is sufficient for photosynthesis of plant to occur efficiently. There by incoming heat flux into the greenhouse enclosure is restricted for rise of inside air temperature due to solar radiation.

The study was conducted in an even span type of greenhouse with effective floor area of 24 m^2 (6m x 4m). The height of the greenhouse wall is 2m where as its center is raised to a height of 3m. A brick wall of 0.275m thick is constructed on the north side of greenhouse. The orientation of greenhouse is from east to west direction. Two vents of each 1m x1m are provided, one on the north and other on the south roof for natural ventilation. There is also provision of fan pad evaporative cooling system in the west side. The south and north roof are covered with jute cloth during daytime in summer periods. For water distribution in the roof surface, a perforated PVC pipe (5 cm. diameter) is provided on the top of the greenhouse. The water after flowing both on the south and north roof is collected in the troughs situated separately in south side and north side of greenhouse as shown in Fig.A. The collected water is returned to the storage sump placed in the northwest corner inside the greenhouse for recirculation.

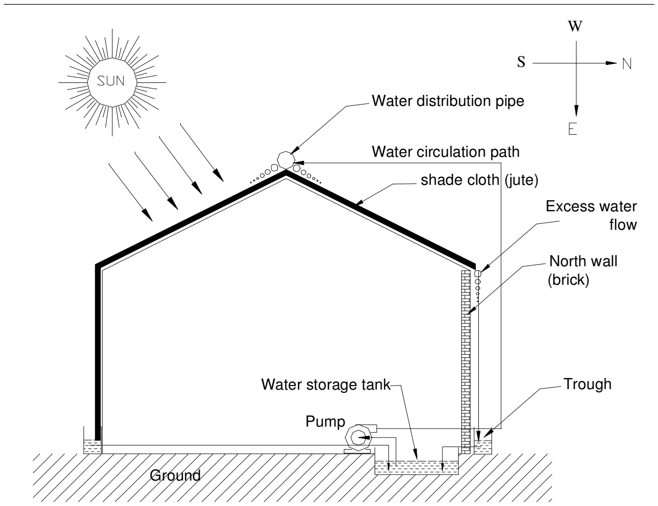

Fig. A : Schematic diagram of water cirulation arrangement over the roofs and south wall of experimental greenhouse

Thermal analysis :

The energy balances of the above system components are expressed as follows.

Flowing water mass on south roof :

$$\dot{m}_w C_w \frac{dT_{wsr}}{dx} dx = [\alpha_j I_{sr}(t) - h(T_{wsr} - T_a) - h_{c,jr}(T_{wsr} - T_r)] b dx \quad ....(1)$$

Solving the above equation in terms of T_{wsr}, the new equation becomes

$$T_{wsr} = f(t)(1-R) + HT_r(1-R) + RT_{wi} \quad ....(2)$$

where,

$$f(t) = \frac{\alpha_j I_{sr}(t) + hT_a}{h + h_{c,jr}}, \; R = \frac{1-e^{-AL}}{AL}, \; A = \frac{(h + h_{c,jr})b}{\dot{m}_w C_w} \text{ and } H = \frac{h_{c,jr}}{h + h_{c,jr}}$$

Flowing water mass on north roof :

$$T_{wnr} = f'(t)(1-R') + H'T_r(1-R') + R'T_{wi} \quad ....(3)$$

where,

$$f'(t) = \frac{\alpha_j I_{nr}(t) + h'T_a}{h' + h'_{c,jr}}, \; A' = \frac{(h' + h'_{c,jr})b}{\dot{m}_w C_w},$$

$$R' = \frac{1-e^{-A'L}}{A'L} \text{ and } H' = \frac{h'_{c,jr}}{h' + h'_{c,jr}}$$

Insulating north wall of greenhouse :

$$\alpha_n\{\Sigma(A_i\tau_i I_i)\}F_n = h_{c,nr}(T_n - T_r)A_n + h_{nb}(T_n - T_a)A_n \quad ....(4)$$

Floor of greenhouse :

$$\alpha_f\{\Sigma(A_i\tau_i I_i)\}(1-F_n) = h_{c,fr}(T_f - T_r)A_f + h_\infty A_f(T_f - T_a) \quad ....(5)$$

Greenhouse room enclosure :

$$\begin{aligned}
&(1-\alpha_n)\{\Sigma(A_i\tau_i I_i)\}F_n + (1-\alpha_f)\{\Sigma(A_i\tau_i I_i)\}(1-F_n) + h_{c,jr} \\
&(T_{wsr} - T_r)A_{sr} + h'_{c,jr}(T_{wnr} - T_r)A_{nr} \\
&+ h_{c,nr}(T_n - T_r)A_n + h_{c,fr}(T_f - T_r)A_f = \\
&0.33NV(T_r - T_a) + h_d A_d(T_r - T_a) \\
&+ \{\Sigma(A_i U_i)\}(T_r - T_a) + M_a C_a \frac{dT_r}{dt}
\end{aligned} \quad .....(6)$$

Rearranging Eq. (4) in terms of $(T_n - T_r)$, the new equation becomes

$$h_{c,nr}(T_n - T_r) = F_1 \frac{\alpha_n\{\Sigma(A_i\tau_i I_i)\}F_n}{A_n} - U_n(T_r - T_a) \quad ....(7)$$

where, $F_1 = \frac{h_{c,nr}}{h_{c,nr} + h_{nb}}$ **and** $U_n = \frac{h_{c,nr}\, h_{nb}}{h_{c,nr} + h_{nb}}$

Likewise rearranging Eq. (5) in terms of $(T_f - T_r)$, the new equation becomes

$$h_{c,fr}(T_f - T_r) = F_2 \frac{\alpha_f\{\Sigma(A_i\tau_i I_i)\}(1-F_n)}{A_f} - U_f(T_r - T_a) \quad ....(8)$$

where, $F_2 = \frac{h_{c,fr}}{h_{c,fr} + h_\infty}$ and $U_f = \frac{h_{c,fr}\, h_\infty}{h_{c,fr} + h_\infty}$

Now rearranging Eq. (2) in terms of $(T_{wsr} - T_r)$, the new equation becomes

$$h_{c,jr}(T_{wsr} - T_r) = h_{c,jr} f(t)(1-R) + h_{c,jr} H(1-R)T_r + h_{c,jr} R T_{wi} - h_{c,jr} T_r \quad ....(9)$$

Similarly rearranging Eq. (3) in terms of $(T_{wnr} - T_r)$ the equation becomes

$$h'_{c,jr}(T_{wnr} - T_r) = h'_{c,jr} f'(t)(1-R') + h'_{c,jr} H'(1-R')T_r + h'_{c,jr} R' T_{wi} - h'_{c,jr} T_r \quad ....(10)$$

Then substituting Eqs. (7), (8), (9), and (10) in Eq. (6) and after rearranging the final equation can be written in the form of

$$\frac{dT_r}{dt} + aT_r = B(t) \quad ....(11)$$

where,

$$a = \frac{1}{M_a C_a}[0.33NV + h_d A_d + \{\Sigma(A_i U_i)\} + U_n A_n + U_f A_f + \{h_{c,jr} H(1-R) - h_{c,jr}\} A_{sr} + \{h'_{c,jr} H'(1-R') - h'_{c,jr}\} A_{nr}]$$

$$B(t) = \frac{F(t) + (UA)_{eff}\, T_a}{M_a C_a}$$

$$F(t) = \{R_1 + R_2 + F_1 \alpha_n F_n + F_2 \alpha_f (1-F_n)\}\{\Sigma(A_i \tau_i I_i)\} + \{h_{c,jr} f(t)(1-R) + h_{c,jr} R T_{wi}\} A_{sr} + \{h'_{c,jr} f'(t)(1-R') + h'_{c,jr} R' T_{wi}\} A_{nr}$$

$$(UA)_{eff} = U_n A_n + U_f A_f + h_d A_d + \{\Sigma(A_i U_i)\} + 0.33NV$$

$$\{\Sigma(A_i U_i)\} = A_e U_e + A_{ww} U_{ww} + A_s U_s + A_n U_n + A_{sr} U_{sr} + A_{nr} U_{nr}$$

$$\{\Sigma(A_i \tau_i I_i)\} = A_e \tau_e I_e + A_{ww} \tau_{ww} I_{ww} + A_s \tau_s I_s + A_{sr} \tau_j I_{sr} + A_{nr} \tau_j I_{nr}$$

$R_1 = (1-\alpha_n)F_n$ and $R_2 = (1-\alpha_f)(1-F_n)$

$F_1 = \frac{h_{c,nr}}{h_{c,nr} + h_{nb}}$ and $F_2 = \frac{h_{c,fr}}{h_{c,fr} + h_\infty}$

In order to obtain the analytical solution of Eq. (11), the following assumptions have been made

– Time interval Δt is small

– Function $B(t)$ is constant *i.e.*, $B(t) = \overline{B(t)}$ for the time interval Δt.

The solution of Eq. (11) can be written as

$$T_r = \frac{\overline{B(t)}}{a}(1 - e^{-a\,t}) + T_{ro} e^{-a\,t}$$

where, T_{ro} is the temperature of greenhouse air at t=0. $T_{wi} \approx T_a$

EXPERIMENTAL FINDINGS AND ANALYSIS

By the help of above thermal analysis, the influence of out side relative humidity, flow rate of water, absorptivity of shading material and length of roof on the cooling effect of greenhouse room air temperature can be very well studied both theoretically and experimentally. But the literature suggests that for the large values of relative humidity, more heat flux filters in through the roof on account of lower evaporation resulting in the attainability of higher water temperatures. Similarly with the increase of flow rate of water, the rate of evaporation of water decreases due to its exposure to sun for fewer periods and there by significant cooling is not achieved inside the greenhouse. Also absorptivity of shading material plays an important role in evaporation of flowing water over the roof. The more the solar radiation is absorbed, the more the evaporation occurs and more cooling effect is felt. With the increase of length of roof, water gets more thermal energy for evaporation by covering a longer distance, as the basic concept behind this study is that evaporation causes cooling.

Appendix A :

$h_o = 5.7 + 3.8v$, Duffie and Backman (1991) [7] (A.1)

$h_i = 2.8 + 3.0v$, Watmuff *et al.* (1977) [8] (A.2)

$$h_c = h_i \quad (A.3)$$

$$h = h_r + h_c + h_e \quad (A.4)$$

$$h' = h'_r + h_c + h'_e \quad (A.5)$$

$$h_r = \varepsilon\,\sigma\left[\frac{(T_{wsr} + 273)^4 - (T_a + 273)^4}{T_{wsr} - T_a}\right] \quad (A.6)$$

$h_e = 16.273 \times 10^{-3}\, h_c \frac{P_{wsr} - \gamma P_a}{T_{wsr} - T_a}$ Cooper (1973) [9] (A.7)

$P_T = \exp\left[25.317 - \frac{5144}{T + 273.15}\right]$ Fernandez and Chargoy (1990) [10] (A.8)

$$h'_r = \varepsilon\,\sigma\left[\frac{(T_{wnr} + 273)^4 - (T_a + 273)^4}{T_{wnr} - T_a}\right] \quad (A.9)$$

$$h'_e = 16.273 \times 10^{-3}\, h_c \frac{P_{wnr} - \gamma P_a}{T_{wnr} - T_a} \quad (A.10)$$

$$U_n = \left[\frac{1}{h_i} + \frac{L_B}{K_B} + \frac{1}{h_o}\right]^{-1} \quad (A.11)$$

$$h_{nb} = \left[\frac{L_B}{K_B} + \frac{1}{h_o}\right]^{-1} \quad (A.12)$$

$$U_{nr} = \left[\frac{L_j}{K_j} + \frac{1}{h'_{c,jr}}\right]^{-1} \quad (A.13)$$

$$U_e = U_{ww} = U_s = U = \left[\frac{1}{h_i} + \frac{1}{h_o}\right]^{-1} \quad (A.14)$$

$$h_{c,nr} = h_i \quad (A.15)$$

$$h_{c,fr} = h_i \quad (A.16)$$

$$h_\infty = \frac{K_g}{L_g} \quad (A.17)$$

$$h_d = \left[\frac{1}{h_i} + \frac{1}{h_o}\right]^{-1} \quad (A.18)$$

$$U_f = \left[\frac{1}{h_{c,fr}} + \frac{1}{h_\infty}\right]^{-1} \quad (A.19)$$

$$U_{sr} = \left[\frac{L_j}{K_j} + \frac{1}{h_{c,jr}}\right]^{-1} \quad (A.20)$$

Appendix B :

Calculation of heat transfer coefficient for inclined surface south roof ($h_{c,jr}$) and north roof ($h'_{c,jr}$) of greenhouse (Tiwari, 2002) [11].

The coefficient of heat transfer due to free convection

$$h_f = \frac{Nu\,K}{X}$$

where,

Nu = Nusselt number (dimensionless)

K = Thermal conductivity of surrounding air (W/m ^{0}K)

X = Characteristic dimension of system (m), = (length+breadth)/2

Again $Nu = C'(Gr\,Pr)^n\,K'$

where, C' and n are constants depending on the geometry of the system K' is the correction factor

$$Gr = \frac{g\,\beta'\,\Delta T\,X^3\,Pr}{\upsilon^2}$$

where g is acceleration due to gravity (m/s^2)

β' = Coefficient of volumetric thermal expansion of surrounding air (^{0}K^{-1})

Gr = Grahof number (dimensionless)

Pr = Prandtl number (dimensionless)

ΔT = Temperature difference between hot surface and surrounding surface (^{0}C)

υ = Kinematic viscosity (m^2/s)

Pr, K and υ are calculated by using the fluid physical properties at average temperature (T_f) of hot surface (T_1) and surrounding air (T_2) *i.e.*, $T_f = (T_1+T_2)/2$

For moderately inclined plane in laminar flow

$$C' = 0.8, n = \frac{1}{4} \quad \text{and } K' = \left[\frac{\cos\theta}{1 + (1 + \frac{1}{Pr^{\frac{1}{2}}})^2}\right]^{\frac{1}{4}}$$

where θ is the inclination of hot surface and the range of laminar flow is from $10^5 < Gr\ Pr > 2x10^7$ and range of turbulent flow is $2x10^7 < Gr\ Pr > 3x10^{10}$

As in this case θ is same for both south roof and north roof *i.e.*, 63^{0}C hence the values of $h_{c,jr}$ and $h'_{c,jr}$ are same.

Symbols :

A -Area, m^2

b -Width of south and north roof of greenhouse, m

C_a -Specific heat of air, J kg^{-1} ^{0}C^{-1}

C_w -Specific heat of water, J kg^{-1} ^{0}C^{-1}

F_n -Solar fraction, dimensionless, decimal

h -Total heat loss coefficient from water film in jute cloth of south roof to ambient, Wm^{-2} ^{0}C^{-1}

h' -Total heat loss coefficient from water film in jute cloth of north roof to ambient, Wm^{-2} ^{0}C^{-1}

h_c -Convective heat transfer coefficient from water film in jute cloth to ambient, Wm^{-2} ^{0}C^{-1}

h_d -Overall heat transfer coefficient from greenhouse to ambient through door, Wm^{-2} ^{0}C^{-1}

h_e -Evaporative heat transfer coefficient from water film in south roof to ambient, Wm^{-2} ^{0}C^{-1}

h_i -Convective heat transfer coefficient from inside of greenhouse to ambient, Wm^{-2} ^{0}C^{-1}

h_o -Convective and radiative heat transfer coefficient from greenhouse to ambient, Wm^{-2} ^{0}C^{-1}

h_r - Radiative heat transfer coefficient from water film in south roof to ambient, Wm^{-2} ^{0}C^{-1}

h_∞ -Heat transfer coefficient from floor to larger depth of ground through conduction, Wm^{-2} ^{0}C^{-1}

h'_e -Evaporative heat transfer coefficient from water film in north roof to ambient, Wm^{-2} ^{0}C^{-1}

h'_r - Radiative heat transfer coefficient from water film in north roof to ambient, Wm^{-2} ^{0}C^{-1}

h_{nb} -Heat transfer coefficient from north wall to ambient, Wm^{-2} ^{0}C^{-1}

$h_{c,jr}$ -Convective heat transfer coefficient from floor to greenhouse air, Wm^{-2} ^{0}C^{-1}

$h_{c,jr}$ -Convective heat transfer coefficient from jute cloth in south roof to greenhouse air, Wm^{-2} ^{0}C^{-1}

$h'_{c,jr}$ -Convective heat transfer coefficient from jute cloth in north roof to greenhouse air, Wm^{-2} ^{0}C^{-1}

$h'_{c,nr}$ - Convective heat transfer coefficient from north wall to greenhouse air, Wm^{-2} ^{0}C^{-1}

$I(t)$ -Solar radiation, Wm^{-2}
K_B -Thermal conductivity of brick, $Wm^{-1}\,{}^0C^{-1}$
K_g - Thermal conductivity of ground, $Wm^{-1}\,{}^0C^{-1}$
K_j - Thermal conductivity of jute cloth, $Wm^{-1}\,{}^0C^{-1}$
L -Length of south and north roof, m
L_B -Thickness of brick north wall, m
L_g -Thickness of ground, m
L_j -Thickness of jute cloth, m
$\dot{m}_w$ -Mass flow rate of water,kg s^{-1}
M_a -Total mass of air in greenhouse, kg
N -Number of air changes per hour
P -Saturated vapour pressure, N m^{-2}
t -Time in second
T -Temperature, ${}^0C$
U -Overall heat transfer coefficient for greenhouse cover, $Wm^{-2}\,{}^0C^{-1}$
U_f -Overall heat transfer coefficient from floor to greenhouse and ground,$Wm^{-2}\,{}^0C^{-1}$
U_n -Over all heat transfer coefficient for north wall, $Wm^{-2}\,{}^0C^{-1}$
U_{nr} -Over all heat transfer coefficient from north roof to greenhouse air and ambient,$Wm^{-2}\,{}^0C^{-1}$
U_{sr} -Over all heat transfer coefficient from south roof to greenhouse air and ambient,$Wm^{-2}\,{}^0C^{-1}$
(UA) -Over all heat loss from greenhouse,W^0C^{-1}
v -Velocity of air,m s^{-1}
V -Volume of greenhouse,m^3

Greek:

α -Absorptivity, dimensionless
σ -Stefan's constant,5.66 x 10^{-8} $Wm^{-2}K^{-4}$
ε -Emissivity, dimensionless
τ -Transmitivity, dimensionless
γ -Relative humidity, decimal
∞ -Infinity

Subscripts:

a -Ambient
d -Door of greenhouse
e -East wall of greenhouse
f -Floor of greenhouse
i -Different walls and roofs of greenhouse
j -Jute cloth
n -North wall of greenhouse
r -Greenhouse room
s -South wall of greenhouse
w -Water
nr -North roof of greenhouse
sr -South roof of greenhouse
wi -Inlet water
ww -West wall of greenhouse
eff -Effective
wnr -Water film in north roof
wsr - Water film in south roof

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