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Research Paper :

Study of subsurface drainage of two layered soil with exponentially declining replenishment rate

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ABSTRACT

The transient drainage for two layered soil, when the initial water level in the ditches is at interface of the layers has been studied. The flow system in the layered soils has been described by Boussinesq's equation in the form of the Girinsky potential. The analytical solutions were derived to describe rise and decline of water table in response to time varying replenishment rate for exponentially declining replenishment rate. The Laplace transformation was used to obtain the solution with initial and boundary conditions. The two layered drainage problem was also investigated on a simulated vertical Hele-Shaw model for the validation of theoretical solutions. The comparison of results of observed and computed water table at mid point were found to be in good agreement for the entire duration of the experiments for exponentially declining replenishment rate. From the comparison, it was also revealed that the solution for layered soil gave more accurate results as compared to homogeneous soil with weighted average hydraulic conductivity (WAHC). Hence, the proposed solutions can be used for the design of drainage system or flow through the aquifer of two layered soil.

Key words : Subsurface drainage, Two layered soil, Drainage, Replenishment rate

Subsurface drainage has a very important effect on agriculture productivity because of draining the excessive water on time. It provides aeration, and prevents water logging and salinization of the plant root zone. Subsurface drainage is a measure, which controls the rise of water table as a function of time and quantity of water to be removed and plays an important role in lowering of water table in case of excess irrigation and recharge due to rainfall..

Most of the drainage theories available in literature attempt to describe water table behaviour in response to uniform percolation related to flat lands and have been developed by obtaining the solution of partial differential equations derived by Boussinesq's equation (1877 and 1904), which is based on Dupuit-Forchheimer assumptions and potential theory. Massland (1959) analysed the problem of water table fluctuation in response to constant recharge, intermittent constant recharge and intermittent instantaneous recharge. Sewa Ram and Chauhan (1987a) obtained transient solutions for water table rise in a sloping aquifer receiving time varying recharge and lying between two parallel ditches reaching up to impermeable layer. The solution was obtained for exponentially declining replenishment rate with time.

Many investigator such as Chieng and Uziak (1991), Sharma *et al.* (1991), Shiv Kumar and Chauhan (1999), and Sharma *et al.* (2000) presented analytical solutions for steady or unsteady state condition of layered soil. Some earlier cited workers also conducted experiments on Hele-Shaw model to verify their theoretical investigations. Some of these are Khan *et al.* (1989), and Yognedra Kumar (1998).

Thus, the objective of the present paper is to develop analytical solution for unsteady state rise and decline of water table under exponentially declining replenishment rate for two layered soil, in the initial water level in the interface of layers.

Problem formulation:

The assumptions considered for formulating the mathematical problem are given below:

– The soil consisted of two layers that were unconfined, homogeneous and isotropic within themselves.

– The phreatic surface lied over a flat impermeable bed.

– Dupuit-Forchheimer assumptions were valid.

– The generalized Boussinesq equation was valid for a stratified aquifer.

– The drainage system consisted of equally spaced open ditch drain reaching upto impervious layer.

–The initial water table was at h_0 for t = 0 and $0 \leq x \leq L$.

The flow system in the drains under unsteady state condition was taking place due to variable replenishment rate. The Boussinesq's equation for unsteady state flow, in the form of Girinsky potential is written as below:

$$\frac{\partial^2\theta}{\partial x^2} = \frac{1}{k'}\frac{\partial\theta}{\partial t} - R \qquad (1)$$

where,

R = Recharge rate per unit surface area

$$k' = \frac{1}{f}\int_o^h K(Z) \cdot dz \qquad (2)$$

The Girinsky potential 'θ' is an integral of all the values of potential at all possible positions of the free surface on the vertical line over 0<z<h, as follows:

$$\theta = \int_o^h (z - h)\; K\; (z) \cdot dz \qquad (3)$$

where,

K (z) = hydraulic conductivity of an individual layer

h = height of phreatic surface above the impervious layer.

Girinsky potential for the upper stratum of the problem under study may be:

$$\theta = K_2 \int_o^{ho} (z-h)\cdot dz + K_1 \int_{ho}^{h} (h-z)\cdot dz \qquad (4)$$

where,

K_1 and K_2 = hydraulic conductivity of the upper and lower layer of the soil

ho = thickness of the lower layer.

Integrating equation (2) within the region under study is written as:

$$k' = \frac{K_2 h_o + K_1 (h_{av} - h_o)}{f_1} \qquad (5)$$

$$h_{av} = h_o + \frac{h_m}{2}$$

where,

f_1 = drainable porosity of the upper layer

h_m = height of water table mid way between the drains above the water levels in the drains

Boundary conditions:

Initial condition

θ (*x*, o) = q_o at all *x* $0 \le x \le L$ at t = 0

Boundary conditions … (6)

θ (o, t) = θ_o at *x* = 0 t > 0

θ (L, t) = θ_o at *x* = L t > 0

The value of θ and θ_o

$$\theta = \frac{-K_1}{2}\left[(h + (C-1)h_o)^2\right] - C(C-1)h_o^2\}$$

$$\theta = \frac{-K_1}{2}\left[(h + B)^2 - d\right] \qquad (7)$$

$$\theta_0 = \frac{-K_1}{2}\left[(h_o)^2\right] \qquad (8)$$

where,

$$d = C(C-1)h_o^2 \qquad B = (C-1)h_o \qquad C = \frac{K_2}{K_1}$$

Analytical solutions:

The partial differential equation in the form of Girinsky potential for exponentially declining replenishment rate, R (t) = R e $^{-rt}$ is written as below:

$$\frac{\partial^2\theta}{\partial x^2} = \frac{1}{k'}\frac{\partial\theta}{\partial t} - R\cdot e^{-rt} \qquad (9)$$

The general solution of equation (11) is written as

$$\bar{\theta}(x,s) = C_1 \cos h\sqrt{s/k'}\; x + C_2 \sin h \cdot \sqrt{s/k'}\; x + \frac{\theta_o}{s} + \frac{Rk'}{s(s+r)}$$

.....(10)

where, C_1 and C_2 are constant of integration

Taking Laplace transformation of boundary value problem using equation (6)

$$\theta\,(o,s) = \theta_o/s$$
$$\theta\,(L,s) = \theta_o/s \qquad (11)$$

By putting these boundary conditions in (10)

$$C_1 = -\frac{Rk'}{s(s+r)}$$

$$C_2 = \frac{\frac{Rk'}{s(s+r)}\left(\cos h\sqrt{s/k'}\; L - 1\right)}{\sin h\; \sqrt{s/k'}\; L}$$

Substitute value of C_1 and C_2 in equation (10) and rearranging the term

$$\bar{\theta}(x,s) = \frac{\theta_o}{s} - \frac{Rk'}{s(s+r)}\left[\frac{\sin h\sqrt{s/k'}\,(L-x)}{\sin h\cdot\sqrt{s/k'}\cdot L} + \frac{\sin h\,\sqrt{s/k'}\;x}{\sin h\,\sqrt{s/k'}\;L} - 1\right] \quad .....(12)$$

By taking inverse transformation the solution of equation (12) may be written as follows:

$$\theta(x,t) = \theta_o - \frac{4Rk'}{\pi}\sum_{n=0}^{\infty}\frac{1}{(2n+1)}\left[\frac{L^2}{(2n+1)^2\pi^2k'+r\cdot L^2}\right]\left(1-e^{-\frac{(2n+1)^2\pi^2k'+r\cdot L^2)t}{L^2}}\right)\cdot\sin\frac{(2n+1)\pi)}{L} \quad (13)$$

By putting values of Girinsky potentials, the general solution of phreatic surface for two-layered soil is expressed by,

$$h = -(C-1)h_o + \sqrt{C(C-1)h_o^2 + Ch_o^2 + \frac{8Rk'}{K_1\pi}\sum_{n=1,3,5}^{\infty}\frac{1}{n}\left[\frac{L^2}{n^2\pi^2k'+r\cdot L^2}\right]\left(1-e^{-\frac{(n^2\pi^2k'+r\cdot L^2)\,t}{L^2}}\right)\cdot\sin\frac{n\pi\pi}{L}} \quad .....(14)$$

The value of h (L/2, t) at mid point between the parallel drain is obtained as

$$h(L/2,t) = -(C-1)h_o + \sqrt{C(C-1)h_0^2 + Ch_0^2 + \frac{8Rk'}{K_1\pi}\sum_{n=1,-3,5}^{\infty}\frac{1}{n}\left[\frac{L^2}{n^2\pi^2k'+r\cdot L^2}\right]\left(1-e^{-\frac{(n^2\pi^2k'+r\cdot L^2)\,t}{L^2}}\right)} \quad (15)$$

For the case, when C = 1 *i.e.* homogeneous and isotropic aquifer equation (14) reduce to

$$h^2 = h_o^2 + \frac{8Rk'}{K\pi}\sum_{n=1,3,5}^{\infty}\frac{1}{n}\left[\frac{L^2}{n^2\pi^2k'+r\cdot L^2}\right]\cdot\sin\frac{n\pi\pi}{L}\left(1-e^{-\frac{(n^2\pi^2k'+r\cdot L^2)\,t}{L^2}}\right) \quad (16)$$

And equation (15) reduce to

$$h^2(L/2,t) = h_o^2 + \frac{8Rk'}{K\pi}\sum_{n=1,-3,5}^{\infty}\frac{1}{n}\left[\frac{L^2}{n^2\pi^2k'+r\cdot L^2}\right]\left(1-e^{-\frac{(n^2\pi^2k'+r\cdot L^2)\,t}{L^2}}\right) \quad (17)$$

METHODOLOGY

The experimental verifications of theoretical solution were conducted on a vertical Hele-Shaw model consisting of two closely parallel plates. It was designed and fabricated in the workshop of College of Technology and assembled in 'Flow Through Porous Media Laboratory' of the Department of Irrigation and Drainage Engineering, Pantnagar. The model was assembled for studies on simulated two layered soils for required boundary conditions.

The exponentially declining replenishment rate was obtained by operating the recharge nozzles at a decreasing head of oil in the channel. The channel was filled with oil up to a certain marked level before the start of the experiment. During the experimental run eighteen equally spaced nozzles were operated till the recharge rate through the nozzles was considerably reduced. The calibration for the recharge rate was done by measuring the volume of outflow from the 18 nozzles used in the experiments. The model was filled with the oil up to the desired height and it was kept undisturbed for 2-3 hours till the oil level in the model reached horizontal and constant level throughout its length. The replenishment rate at the start of the experiment *i.e.* t = 0, was taken equal to the recharge rate at constant head. An exponential relationship of the form $R(t) = R\exp^{-rt}$ was obtained as shown in Fig. 1.

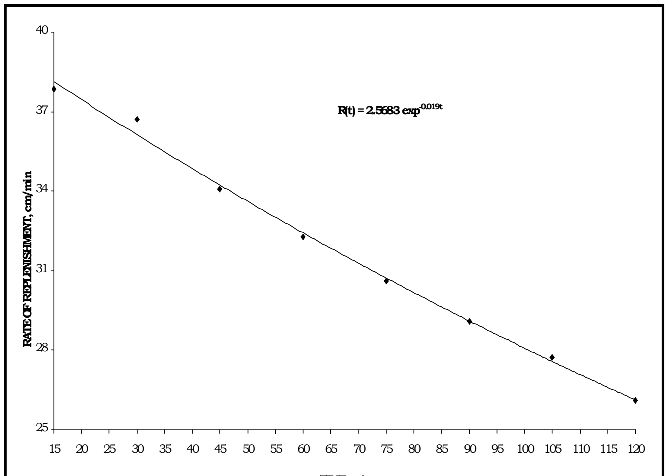

Fig. 1 : Exponentially declining replenishment rate

RESULTS AND DISCUSSION

The hydraulic conductivity of model of upper layer was estimated to be 35.32 cm/min and for lower layer as 10.36 cm/min, which provided K_2/K_1 ratio as 0.29. The ratio of replenishment rate to hydraulic conductivity (R/K_1) was estimated to be 0.0349.

Unsteady state rise of water table in response to exponentially declining replenishment rate:

The observed and computed water table profiles are shown in Fig. 2. The rising water table at mid point for layered soil and with weighted average hydraulic conductivity are shown in Fig. 3 and 4, respectively.

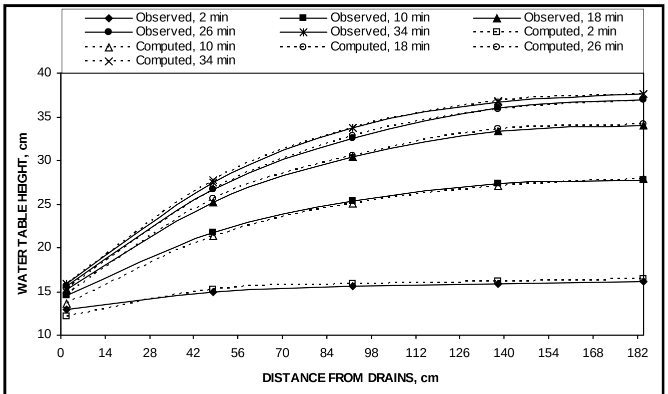

Fig. 2 : Observed and computed rising water table profiles for exponentially declining replenishment rate

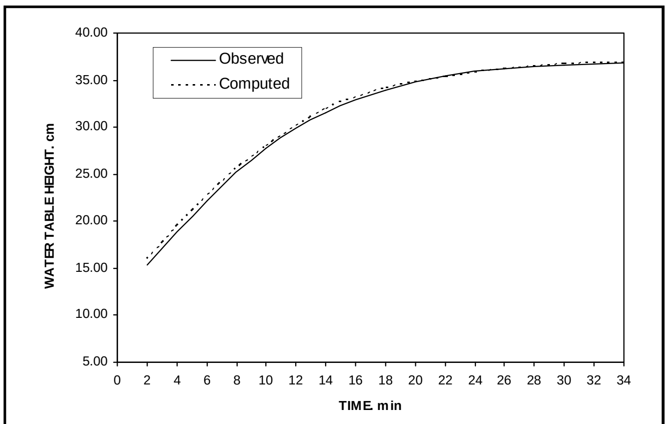

Fig. 3 : Rising water table at mid point for layered soil with exponentially declining eplenishment rate

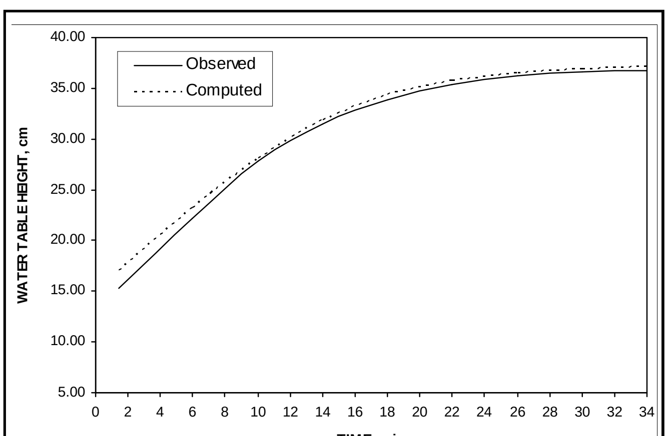

Fig. 4 : Rising water table at mid point for weighted average hydraulic conductivity with exponentially exponentially declining replenishment rate

The average deviation in the initial stage of rising of water table near the ditches was 4.14% and at mid point 0.87%. The deviation near the ditches was larger and decreased with time. The deviation at mid point for layered soil ranged from 0.30 to 0.95% and corresponding to weighted average hydraulic conductivity ranged from 0.85 to 3.6%. Initially at mid point, the deviation was more and decreased with increase in time in layered soil as well as with weighted average hydraulic conductivity in homogeneous case.

Unsteady state decline of water table in response to exponentially declining replenishment rate:

The variation between observed and computed water table profiles is shown in Fig. 5. The water table at mid point for layered and for weighted average hydraulic conductivity is shown in Figs. 6 and 7, respectively.

The average deviation near the ditch was found to be 6.93% and at mid point 0.73%. It was observed that the deviation in the initial stage of declining of water table was considerably less and increased with increase in time. The deviation at mid point for layered soil ranged from 0.08 to 1.5% and with weighted average hydraulic conductivity ranged from 0.49 to 1.86%. At mid point of lowering water table, the deviation was lesser as compared to near the ditch.

Sewa Ram and Chauhan (1987b) also conducted experiments for the homogeneous isotropic sloping aquifers on a vertical Hele-Shaw model and compared the results with his proposed analytical solution. It was reported that deviation at the mid point during declining water table was found to be within 2.48 to 7.49%. It was also reported that more deviation observed near the ditches. A similar trend of deviation was observed in the present study of two layered soils and error was found to be within the similar permissible range. Thus, the proposed

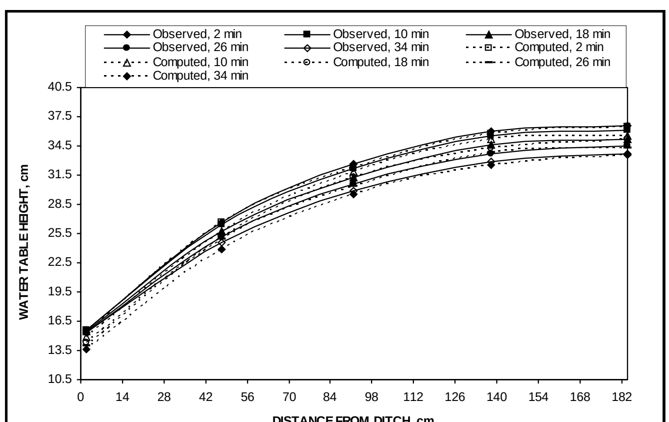

Fig. 5 : Observed and computed falling water table profiles for exponentially declining eplenishment rate

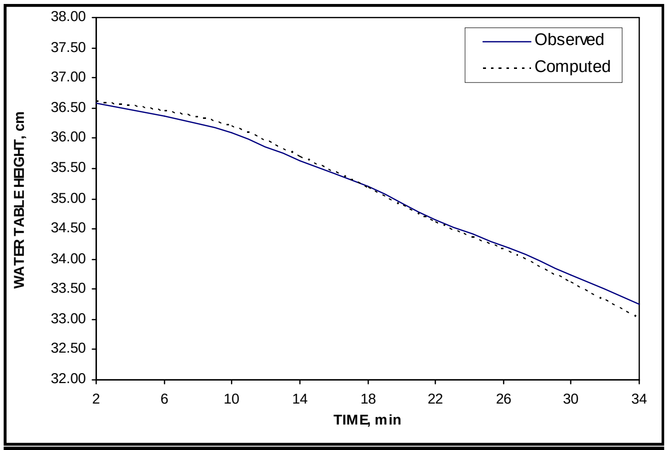

Fig. 6 : Falling water table at mid point for layered soil withexponentially declining replenishment rate

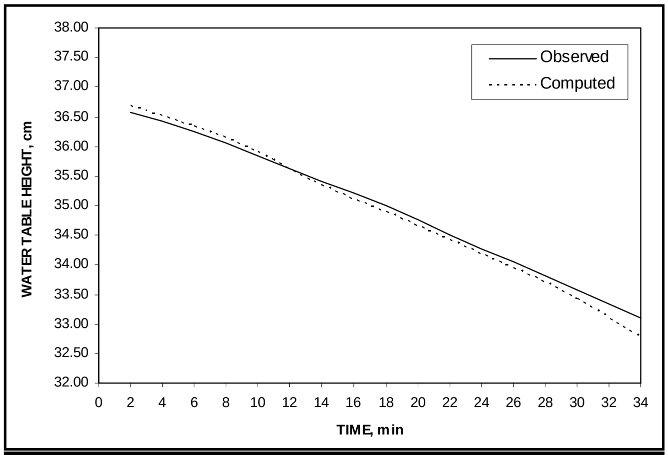

Fig. 7 : Falling water table at point for weighted average hydraulic conductivity withexponentially declining replenishment rate

analytical solutions predicted reasonably correct water table close to the experimental results in two layered soil.

Conclusion:

The problem of transient drainage for two layered soil, when the initial water level in the ditches is at interface of the layers, has been studied. The flow system in the layered soils has been described by Boussinesq's equation in the form of the Girinsky potential.The analytical solutions for unsteady state rised and declined of water table for two layered soil, with exponentially declining replenishment rate compared with laboratory experiments on simulated viscous fluid vertical Hele-Shaw model. The model fluid was used HP 140 oil with kinematic viscosity of 8 stokes at 28°C. The proposed mathematical models for layered soils are considerably accurate for practical use in the design of subsurface drainage systems. The following conclusions were drawn in this study.

The deviation of rising water table for exponentially declining replenishment rate was less at mid point as compared to near the ditch and decreased with increase in time. The deviation of falling water table for exponentially declining replenishment rate was less at mid point as compared to near the ditch and increased with increase in time. The deviation for exponentially declining replenishment rate of solution for homogeneous soil with weighted average hydraulic conductivity at mid point was found to be more in comparison to the solution for layered soil. It reveals that, the solution for layered soil gave relatively accurate results as compared to the solution for homogeneous soil with weighted average hydraulic conductivity. The error at mid point between ditches for rising and falling water table for exponentially declining replenishment rates were found to be within the reasonable limits for a practical use of the proposed analytical solutions under an unsteady state condition.

Notations:

θ = Girinsky potential function

θ (0, t) = Girinsky potential at x = 0

θ (L, t) = Girinsky potential at x = L

b = spacing between two plates, L

f = Drainable porosity, dimensionless

L = Spacing between drains, L

r = Coefficient in the time varying exponentially declining replenishment rate, T^{-}

R (t) = Time varying replenishment rate per unit surface area of the aquifer, LT^{-1}

s = parameter of the Laplace transformation

t = Time, T

x = horizontal distance from a reference point L

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