RESEARCH PAPER *Asian Science* (June & December, 2011) Vol. 6 Issue 1 & 2 : 58-62

K-Theory for bisological processes of infinite C*-algebra

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ABSTRACT

Extensions algebras of unital purdy infinite simple c* algebras have been studied on the vast canvas of complex bisological processes. K-Theory is developed to understand the impact of such processes on the global, astronomical and cosmic scale.

Kumar, Deepak and Bajaj, M.M. (2011). K-Theory for bisological processes of infinite C*-algebra. *Asian Sci.*, **6**(1 & 2): 58-62.

Key Words : K-Theory, Bisological processes, Multi-cellular organisms, Infinite body disorders, Innumerable diseases, C*-algebra, Extension, Purely infinite

INTRODUCTION

In the 1960s, Attiyah and Hirzebruch developed the K-theory which is based on the work of Grothendieck in algebric geometry. It was introduced as a tool in C* algebras theory in the early 1970s through some specific important applications. One is the classification of AF-algebras given by Elliott (1). Today K-theory is an active research area and

– An useful tool for the study of C* algebras of complex BIS processes. (Fig. 1-4)

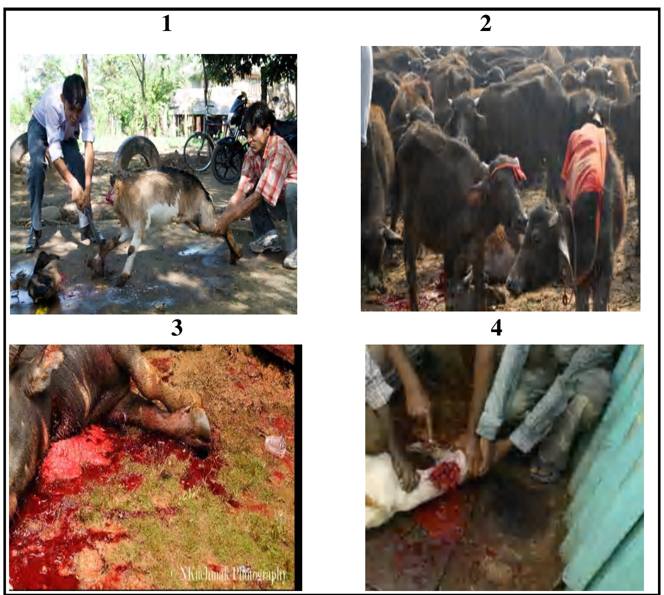

Fig. 1 - 4: C* algebras of complex BIS processes

– K-theory is very useful in non commutative geometry

– Algebric topology of the neural, cellular, viral and bacterial assemblies.

– Nanotechnology and viral proliferation.

Let A be a C* -algebra, and let p, q be projections in A. We write p~q if they are (Murrary-von Neumann) equivalent i.e. p=v*v and q=vv* for some partial isometry v in A. We denote the Murrary-non Neumann equivalence class containing p by [p]. Write $p \prec q$, if p is equivalent to a subprojection of q.

The relations are also defined in the matrix algebras on A.

A projection of p in a c* -algebra A is said to be infinite, if it is equivalent to a proper subprojection of itself.

If p is non-zero and if there are mutually orthogonal projections p_1, p_2 in A such that p_1+p_2 $p_2 \leq p$ and $p \sim p_1 \sim p_2$, then p is called properly infinite. A nonzero projection p is properly infinite, if and only if $p \oplus p \prec p$.

A unital C*-algebra A is called properly infinite, if its unit 1_A is a properly infinite projection.

Infinite C*-algebra:

A unital simple C*-algebra A, which is not isomorphic to $\square_3$ is called purely infinite, if for every non-zero positive element a 1in A, there is an element x in A such that x*ax=1.

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Let A and B be C* -algebras and assume that B is stable. An essential extension of A by B is a short exact sequence of C* -algebras:

$$0 \rightarrow B \xrightarrow{i} E \xrightarrow{q} A \rightarrow 0,$$

where E is a C*-algebra which contains B as an essential ideal. There are unique injective homomorphisms σ: E → M(B) and τ : A → M(B)/B, the Busby invariant, such that

$$\begin{array}{ccccccccc} O & \rightarrow & B & \xrightarrow{i} & E & \xrightarrow{q} & A & \rightarrow & 0 \\ & & \downarrow & & \downarrow & & \downarrow & & \\ O & \rightarrow & B & \xrightarrow{i} & M(B) & \xrightarrow{q} & M(B)/B & \rightarrow & 0 \end{array}$$

commutes,

where M(B) is the multiplier algebra of B. We assume that the extension is in standard form, that is i and s are inclusion maps, and hence $E=\pi^{-1}(\tau(A))$, we called it an extension algebra of A by B.

Compact operators:

In recent years, many have studied the extension algebras of certain C*-algebras 4, 6, 7, 10-13, 17, 18/ In [14] and [15], K-theory of extension algebras of purely infinite simple C*-algebras by compact operators is described. In [17], the C*-algebras obtained by extension of a unital purely infinite simple C*-algebra by a purely infinite simple stable C*-algebra was considered. In this paper, K-theory of such C*-algebras by using the results of [17] is described. The results may be useful for classifying the extension algebras of certain C*-algebras.

In this paper, an ideal of a C*-algebra is always a closed two-sided ideal. For a unital C*-algebra A, we let U(A) denote the group of unitary elements in A, U^0 (A) denotes its connected component containing the unit of A, and $U_n(A) = U(M_n(A))$ and $U_n^0(A)= U^0(M_n(A))$ for each positive integer n.

RESEARCH FINDINGS AND ANALYSIS

Let A and B be purely infinite simple C*-algebras, and suppose that A is unital and B is stable. Let

$$0 \rightarrow B \rightarrow E \rightarrow A \rightarrow 0$$

be a unital essential extension of A by B, where E is a unitel C*-algebra.

It is proved in [17] that every non-zero projection in E is properly infinite.

Every σ-unital purely infinite simple C*-algebra is either unital or stable [23]

Conditions satisfied by projection set P:

Let A and B be purely infinite simple C*-algebras, and suppose A is unital and B is stable, and let E be an extension algebra of A by B. Let P be the set of all projections in E/B. Then P satisfies the following conditions:

($\prod_1$) : If p, q ε P and pq=0, then p+q ε P.

($\prod_2$) : If p ε P and p' is a projection in A such that p~p', then p' ε P

($\prod_3$) : For any p, q ε P, there is p' ε P such that p'~p, p'<q, and q-p'ε P

($\prod_4$) : If q is a projection in A, and there is a projection p ε P such that p ≤ q, then qε P.

Proof:

Claim, p ε P if and only if 1 ~ p.

If p ε P, then p is a full properly infinite projection. It is well-known that there exists a positive integer n and $x_1, x_2, ... x_n$ in E such that $1=\sum_{i=1}^{n} x_i p x_i^*$. It follows that

1~p⊕p⊕....⊕ p~p. Conversely, if 1~p, then p if full, and hence p ε P. The claim is proved.

By the Claim, $\prod_1$, $\prod_2$, $\prod_4$ are trivial. We only need to show $\prod_3$. For any p, q e P, since q is properly infinite, there are projections q_1, q_2 £ q such that $q_1 \sim q_2 \sim q$ and $q_1q_2 = 0$. So $1 \sim q_1$, and hence $p \sim q_1$. Therefore, there is a projection p'£q_1 such that p ~ p'. Since

q_2 ε q - q_1 ε q = p'.

and q_2 e P, we have q –p' e P

Let A and B be purely infinite simple C*-algebras and suppose that A is unital and B is stable, and let E be an extension algebra of A by B. Then

$K_0(E)$ ={[p]|peP}

In particular, for p, q e P, if [p] = [q] in $K_0(E)$, then p~q.

Proof. It follows by Theorem 2.1 and Theorem 1.4 of [2].

For every short exact sequence of C*-algebras

$$0 \rightarrow B \rightarrow E \xrightarrow{\pi} A \rightarrow 0$$

There is a six-term exact sequence of K-theory:

$$\begin{array}{ccc} K_0(B) \rightarrow K_0(E) \rightarrow K_0(A) \\ \uparrow \qquad\qquad\qquad \downarrow \\ K_1(A) \leftarrow K_1(E) \leftarrow K_1(B) \end{array}$$

A unital C*-algebra A of complex BIS processes has real rank zero. It is written as RR(A)=0, if the set of invertible self-adjoint elements is dense in the set of self-adjoint elements of the vast number of abattoirs in A. A purely infinite simple C*-algebra of complex BIS event has real rank zero.

Four equivalent statements of BIS types of C* systems:

Let A and B be purely infinite simple C*-algebras, and suppose that A is unital and B is stable, and let E be an unital essential extension algebra of A by B. Then the following statements are equivalent.

RR(E) =0

Every projection in A lifts to a projection in E.

π_* 0:K_0(E) $\rightarrow$ K_0(A) is surjective.

δ_0=0.

Proof.

(i)$\Leftrightarrow$(ii) is well-known since RR(A)=RR(B)=0 (Theorem 3.13 of [1]).

(iii)$\Leftrightarrow$(iv) is trivial by the six-term exact sequence of K-theory

(ii) $\Rightarrow$ (iii). For any y ε K_0 (A), y=[p] for some nonzero projection in A. By (ii), there is a projection pεE/B such that p=π(p). Then y=[p] = [π(p)]=π_{*0}([p]).

(iii) $\Rightarrow$ (ii). Let p be a projection in A, and ≠ 0,1. Since π_{*0} is surjective, there is a projection p ε E/B such that [p] = π_{*0} ([p]) = [π(p)]. Then p ~ π(p), and hence p lifts by Lemma 2.8 of [19] (see also Proposition 3.15 of [1]).

BIS projections in real systems involving abattoirs:

Let A be a unital C*-algebra, and let p, q be projections in A. p and q are unitarily equivalent if there is unitary u ε A such that upu*=q. We denote the unitarily equivalence class containing p by $[p]_u$. We say that p and q are homotopic, denoted by p ~ h, q, if p are in the same path component of projection in A. We denote the homotopy class containing p by [p]h.

p ~ h q, if and only if there exists a unitary u ε U_0(A) such that upu=q.

'p~h q' implies 'p~q'. But 'p~q' does not imply 'p~h q' in general. However, for certain C*-algebras, p ~ q if and only if p ~ h q provided that p and q are projections of neither 0 nor 1. AF algebras and purely infinite simple C*-algebras are examples of such C*-algebras [20]. For the case of extension algebras, we have the following results.

Let A and B be purely infinite simple C*-algebras, and suppose that A is unital and B is stable. Let

0→B→E→A→0

be a unital essential extension. Suppose that RR(E) = 0. Let p and q be projections in P such that 1-p, 1-q ε P. Then p ~ q if and only if p ~ h q.

Proof. Suppose p ~ q. By Theorem 3.2 of [21], there exist projections p_1 ≤ q and p_2≤1-q such that p ~ h p_1 + p_2. Since 1-p ε P and p is unitarily equivalent to p_1+p_2, either q - p_1 ε P or 1 – q - p_2 ε P. We assume that q-p1 ε P.

If p_2 ε P, then by Theorem 2.1, there is projection p'_2≤q-p_1 in P such that p_2 ~ p'_2. Since p_2 is orthogonal to p'_2, p_2 ~h p'_2, we use the well-known fact that two mutually orthogonal projections are equivalent if and only if they are homotopic. Moreover, the path of projections joining p_2 and p'_2 can be taken in (1-p_1) E (1-p_1). Hence, p_1+p_2 ~ h. P_1 + P'_2. Since 1-q ε P and p_1 + P'_2 ε P, by Theorem 2.1, there is a projection q_1 ≤ 1-q such that q' ~ p_1 +p'_2. Since q' is orthogonal to p_1 + p'_2, we have q'~h P_1+P'_2, Then

q ~ p ~ p_1 + p_2 ~ p_1 + p'_2 ~ q'

and q is orthogonal to q'. Hence q ~ h q'.

Therefore,

p ~ h p_1 + p_2 ~ h p_1 + p'_2 ~ h q' ~ h q.

If p_2 ε B, then p_1 ε P and 1-q –p_2 ε P. The arguments are similar.

Let A and B be purely infinite simple C*-algebras, and suppose that A is unital and B is stable. Let

0 → B → E → A → 0

be a unital essential extension. Then, for any projection p ε P, there is a projection q ε P with 1 – q ε P such that p ~ q

Proof. Since p ε P, by Theorem 2.1, there is a projection q ε P such that p ~ q £ p and p – q ε P. Then 1 – q = (1-p) + (p-q) ε P

Lemma 2.6. Let A and B be purely infinite simple C*-algebras, and suppose that A is unital and B is stable. Let

0 → B → E → A → 0

be a unital essential extension. Then for any projections p, q e P with 1-p, 1-q ε p, p ~ q if and only if p $\sim_u$ q.

Proof. If p~q, then [p] =[q] in K_0(E), and hence [1-p] = [1-q]. Since 1-p, 1-q ε p, by Corollary 2.2, we have 1-p ~ 1 – q. It follows that p and q are unitarily equivalent.

Theorem Let A and B be purely infinite simple C*-algebras and suppose that A is unital and B is stable. Let

O→B→E→A→O

be a unital essential extension. Then

$K_0(E)$ = {[p]|p,1-p ε P] = {$[p]_u$|p, 1-p ε P}

Furthermore, if RR(E)=0, then

$K_0(E)$ = $[[p]_h$|p, 1-p ε P}

Proof. The first two equations follow by Corollary 2.2, Lemma 2.5 and Lemma 2.6. The third equation is obtained from Lemma 2.4.

Proposition in bisology:

Let A and B be purely infinite simple C*-algebras of complex BIS phenomena suppose A is unital and B is stable.

Let

O→B→E→A→O

be a unital essential extension. Suppose that RR(E)=0, Then for any positive integer n,

$$K_1(E) \cong U(E)/U^0(E) \cong U_n(E)/U_n^0(E)$$

Proof. For each positive integer n, let j_n: → $K_1(E)$ be the natural map. j_n: is surjective since E is properly infinite.

Sequence of slaughtering and murdering of animals, birds and fisheries (Aquatic creatures):

If the extension algebra E has real rank zero, then by the six term exact sequence of K-theory, we have the following exact sequence:

$$0 \to K_1(B) \to K_1(E) \to K_1(A) \xrightarrow{\delta} K_0(B) \to K_0(E) \to K_0(A) \to 0$$

and hence we have the following two short exact sequences:

$$0 \to K_1(B) \to K_1(E) \to K_1(A)/\ker\delta_1 \to 0$$

$$0 \to K_0(B)/\operatorname{Im}\delta_1 \to K_0(E) \to K_0(A) \to 0$$

A unitary u in A lifts to a unitary in E if and only if δ_1([u])=0 in $K_0(B)$, then

Ker δ_1= {[u] ε $K_1(A)$|u ε U(A) and u lifts to a unitary in E}

we also have

Imδ_1= {[p] ε $K_0(B)$|p ε B and [p] = o in $K_0(E)$]
={[τ(u)] ε $K_1(M(B)/B)$|u ε U(A)}

M(B)/B is a unital purely infinite simple C*-algebra since B is a stable purely infinite simple C*-algebra and $K_1(M(B)/B) \cong K_0(B)$.

Discussion:

Extensions algebras of unital purely infinite simple C* algebras have been studied on the vast canvas of complex bisological processes. K-Theory helps us to understand the impact of such processes on the (i) global (ii) astronomical and (iii) Cosmic scales.

Conclusion:

Extension algebras of unital infinite simple c* systems of complex BIS processes explain.

The innumerable diseases occuring in multi-cellular organisms,

Oceanic catastrophics occuring throught the globe.

Origin of bacteria and

Birth of viruses from living state forced annihilation operators (LSFAO) and

Proliferation of viriods from on location to the other due to inhuman and cruel processes being administered by man in the meaningless hope of proteins, hormones, vitamins and good heath.

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Received: May, 2011; Accepted : October, 2011