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RESEARCH PAPER

An inventory model for time dependent deteriorating items with holding cost as a quadratic function of time

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ABSTRACT

This paper deals with an inventory model in which shortages are allowed and partially backlogged. It is also assumed that the demand is a function of stock and selling price and the money value is subject to inflation.

Key Words : Inflation, Stock-dependent and selling price dependent demand, Deterioration, Shortages, Partial backlogging.

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In classical inventory models, researchers ignored deterioration in their models and they also assumed the demand rate to be constant. But in realistic situation, deterioration of items is an important factor in inventory that cannot be disregarded. There are many items in which deterioration depends on fluctuations of weather conditions, humidity, temperature, transportation etc. Further, no one can ignore the price sensitive nature of the demand. For example, in the retail industry, organizations may dynamically adjust their prices in order to boost demand and enhance revenues. More extensive reviews on price dependent demand rate were given by Eliashberg and Steinberg (1991), Gallego and Ryzin (1994). Thus, a more general and realistic system is one which considers the demand as a function of selling price. Deb and Chaudhari (1986) derived inventory models with time dependent deterioration rate. Aggarwal and Hashani (1991) developed a model for deteriorating items in decline market without shortages and production rate was known but varied from one period to another. Gupta and Aggarwal (2000) presented an order level inventory model with time dependent deterioration, demand as a linear function of time and replenishment rate dependent on demand function. Sharma and Kumar (2000) carried out a simple study on deterministic production inventory model for deteriorating items with an exponential declining demand. Yang and Wee (2003) considered a multi-lot-size production inventory system for deteriorating items with constant production and demand rate. Sugapriya and Jeyaraman (2008) discussed an EPQ model for non-instantaneous deteriorating items in which production and demand, both were constant and the holding cost varied with time.

An inventory model is developed for time dependent deteriorating items and the demand is taken to be price dependent. The production runs with constant rate and holding cost varies with quadratic time function. Numerical examples are presented to demonstrate the developed model and to illustrate the procedure.

Assumptions and notations:

– The demand rate R(t) is taken to be selling price dependent and is given by $R(t) = \alpha$ and βs; α and β are positive constants and 's' is selling price per unit.

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– Production rate p (t) per unit time is constant, *i.e.* p (t) = p.
– Deterioration of the units is time dependent.
– Inventory holding cost per unit time is with a_1, a_2, a_3 as constants.
– ‘r’ is the price discount per unit cost.
– ‘A’ is the set up cost and ‘k’ is the production cost per unit.
– q (M) is the maximum inventory level of the product.
– There is no repair or replenishment of deteriorated units.
– Shortages are not allowed and lead time is zero.
– The model has been developed for a finite planning horizon.
– The replenishment occurs instantaneously at an infinite rate.
– Once the production is terminated, deterioration starts and then the price discount is considered.
– $q_1(t)$ is the inventory level of product during the production *i.e.* $0 = t = T_1$.
– $q_2(t)$ is the inventory level for the period when production stops *i.e.* $0 = t = T_2$.
– ‘T’ is the optimal cycle time, ‘T_1’ is the production period and ‘T_2’ is the time during which there is no production of the product, *i.e.*, $T_1 = T - T_2$.
– ‘T_c (T)’ is the total average cost per unit time.

RESEARCH METHODOLOGY

In the beginning the inventory level is zero. Between time $[0,T_1]$ the production and demand start simultaneously and the inventory level is maximum in that interval. After time T_1 production is zero but the demand remains same and deterioration consider in that time period. Production again runs when inventory tends to zero.

The differential equations are :

$$\frac{dq_1(t)}{dt} = p(t) - R(t); 0 \le t \le T_1 \qquad .....(1)$$

$$\frac{dq_2(t)}{dt} + \theta tp2(t) = -R(t); 0 \le t \le T_2 \qquad .....(2)$$

Initial conditions are

$q_1(0) = 0,\ q_2(T_2) = 0$

Solution of equation (1) and (2) are

$$q_1(t) = [p - (\alpha - \beta s)]t; 0 \le t \le T_1 \qquad ....(3)$$

$$q_2(t) = (\alpha - \beta s)\left[(T_2 - t) + \frac{\theta}{6}\left(T_2^3 - t^3\right)\right]e^{-\frac{\theta t^2}{6}}; 0 \le t \le T_1 \qquad ....(4)$$

Cost of production is

$$C_p = pk\frac{T_1}{T} \qquad ....(5)$$

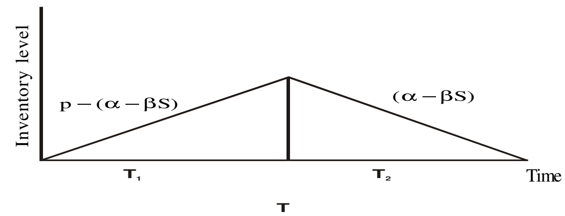
Cost of set up is

$$C_s = \frac{A}{T} \qquad .....(6)$$

Cost of holding is

$$C_h = \frac{1}{T}\left[\int_o^{T_1}\left(a_1 + a_2 + a_3 t^2\right)q_1(t)dt + \int_o^{T_2}\left(a_1 + a_2 + a_3 t^2\right)q_2(t)dt\right]$$

$$= \frac{1}{T}\left[\begin{array}{l}\{p - (\alpha - \beta S)\}\left\{a_1\frac{t_1^2}{2} + a_2\frac{t_1^3}{3}a_3\frac{t_1^4}{4}\right\} \\ + (\alpha - \beta S)\left(a_1\left\{\frac{t_2^2}{2} + a_2\frac{\theta t_2^4}{12}\right\} + a_2\left\{\frac{t_2^3}{6} + \frac{\theta t_2^5}{40}\right\} + a_3\left\{\frac{t_2^4}{12} + a_2\frac{\theta t_2^6}{90}\right\}\right)\end{array}\right] \qquad ..... (7)$$



Representation of inventory model

Cost of deterioration is

$$C_D = \frac{k}{T}\left[q_2(0) - \int_o^{T_2}(\alpha - \beta S)dt)\right] = \frac{k}{6T}(\alpha - \beta S)\theta T_2^3\ . \qquad ....(8)$$

Discount price is

$$D_p = \frac{kr}{T}\int_o^{T_2}(\alpha - \beta S)dt = kr(\alpha - \beta S)\frac{T_2}{T} \qquad .....(9)$$

Therefore the total average cost is given by

$$T_C = C_p + C_S + C_h + C_D + D_P \qquad ....(10)$$

Substitue

$$T = \frac{pT_1}{(\alpha - \beta S)};\ T_2 = \left[\frac{p - (\alpha - \beta S)}{p}\right]T$$

We get

$$T_c = pk\left[\frac{\alpha - \beta s}{p}\right] + \frac{A}{T} + \frac{1}{T}\left[\left(\{p - (\alpha - \beta s)\}\left\{a_1\frac{(\alpha - \beta s)^2}{p^2}\frac{T^2}{2} + a_2\frac{(\alpha - \beta s)^3}{p^3}\frac{T^3}{3} + a_3\frac{(\alpha - \beta s)^4}{p^4}\frac{T^4}{4}\right\}\right)\right.$$

$$+(\alpha - \beta s)\left(\frac{a_1}{2}\left\{\left(\frac{p - (\alpha - \beta s)}{p}\right)^2\right\}T^2 + \frac{a_2}{6}\left\{\left(\frac{p - (\alpha - \beta s)}{p}\right)^3\right\}T^3\right.$$

$$+\left\{\frac{a_3}{12} + \frac{a_1\theta}{12}\right\}\left\{\left(\frac{p - (\alpha - \beta s)}{p}\right)^4\right\}T^4 + \frac{a_2\theta}{40}\left\{\left(\frac{p - (\alpha - \beta s)}{p}\right)^5\right\}T^5$$

$$\left.\left.+\frac{a_3\theta}{90}\left\{\left(\frac{p - (\alpha - \beta s)}{p}\right)^6\right\}T^6\right)\right]$$

$$+\frac{k}{T}(\alpha - \beta s)\frac{\theta}{6}\left\{\frac{p - (\alpha - \beta s)}{p}\right\}^3 T^3 + \frac{kr}{T}(\alpha - \beta s)\left\{\frac{p - (\alpha - \beta s)}{p}\right\}T \qquad ...(11)$$

Now optimize the total cost

$\frac{dT_C}{dT}=0$ and $\frac{d^2T_C}{dT^2}>0$ *i.e.* second derivative is found to be positive ....(12)

Table 1: Relation between β and 'TC'				
β	T_1	T_2	T	TC
0.30	0.33415140	1.362049	1.6962	884.6796
0.35	0.33340155	1.363298	1.6967	882.8362
0.40	0.33265120	1.364549	1.6972	880.9924
0.45	0.33190035	1.365800	1.6977	879.1482
0.50	0.33114900	1.367051	1.6982	877.3036
0.55	0.33041660	1.368383	1.6988	875.4585
0.60	0.32966420	1.369636	1.6993	873.6129
0.65	0.32891130	1.370889	1.6998	871.7670
0.70	0.32817720	1.372223	1.7004	869.9206
0.75	0.32742325	1.373477	1.7009	868.0737

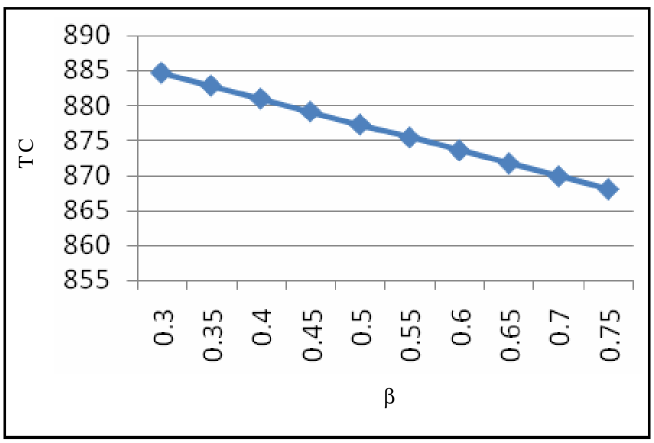


Fig. 1 : With increase in the value of 'p', the total cycle time decreases and the total cost of the system increases

Table 2: Relation between α and 'TC'				
α	T_1	T_2	T	TC
8.00	0.2720250	1.482975	1.7550	728.1757
8.50	0.2867205	1.450980	1.7377	765.7697
9.00	0.3014550	1.421145	1.7226	803.1444
9.50	0.3162760	1.393324	1.7096	840.3173
10.00	0.3311490	1.367051	1.6982	877.3036
10.50	0.3461425	1.342358	1.6885	914.1165
11.00	0.3612215	1.318879	1.6801	950.7680
11.50	0.3764250	1.296575	1.6730	987.2684
12.00	0.3917450	1.275255	1.6670	1023.6270
12.50	0.4072145	1.254886	1.6621	1059.8530

Numerical example :

p = 50 units/unit time; α = 10 units / unit time; r = 0.02/ unit; A = 200 units/set up; θ = 0.20

S= 0.50 units / unit time; $[a_1, a_2, a_3] = [3, 2.5, 2]$; k= 70/ unit time. β = 0.50 units / unit time

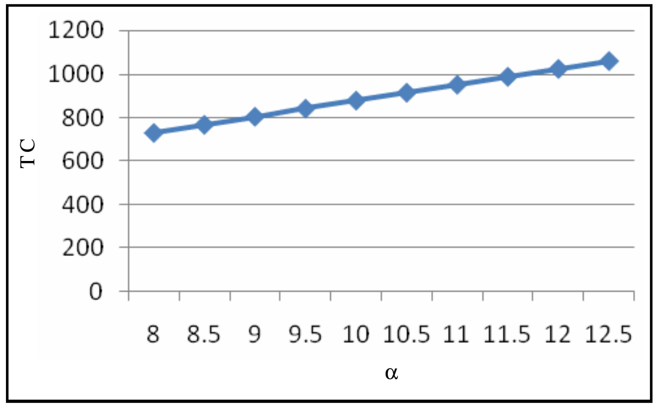


Fig. 2 : With increase in the value of 'p', the total cycle time decreases and the total cost of the system increases

Table 3: Relation between ρ and 'TC'				
ρ	T_1	T_2	T	TC
30	0.638755000	1.326645	1.9654	851.5291
35	0.519396429	1.345104	1.8645	860.4935
40	0.436946250	1.355654	1.7926	867.4062
45	0.376848333	1.362452	1.7393	872.8764
50	0.331149000	1.367051	1.6982	877.3036
55	0.295283182	1.370417	1.6657	880.9559
60	0.266386250	1.372914	1.6393	884.0183
65	0.242625000	1.374875	1.6175	886.6218
70	0.222745714	1.376454	1.5992	888.8619
75	0.205868000	1.377732	1.5836	890.8092

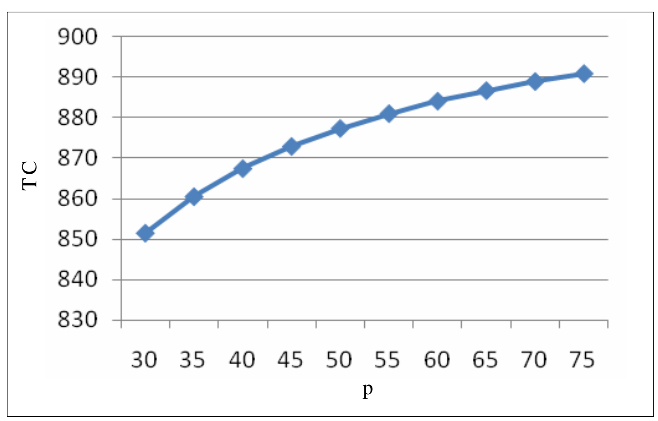


Fig. 3 : With increase in the value of 'p', the total cycle time decreases and the total cost of the system increases

Table 4: Relation between θ and 'TC'

θ	T_1	T_2	T	TC
0.12	0.3648645	1.506236	1.8711	861.5564
0.14	0.3551145	1.465986	1.8211	865.801
0.16	0.3463395	1.429761	1.7761	869.823
0.18	0.338403	1.396997	1.7354	873.6497
0.20	0.331149	1.367051	1.6982	877.3036
0.22	0.3244995	1.339601	1.6641	880.8030
0.24	0.3183765	1.314324	1.6327	884.1635
0.26	0.312702	1.290898	1.6036	887.3983
0.28	0.307398	1.269002	1.5764	890.5187

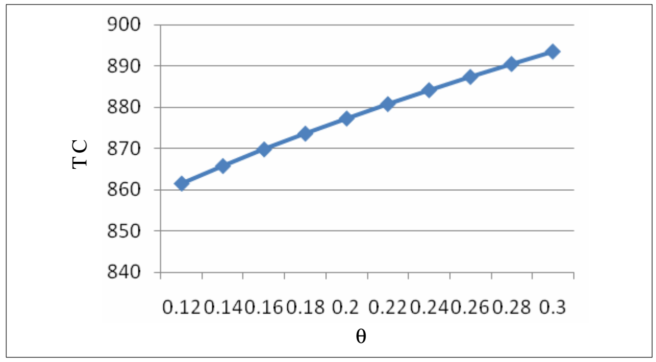


Fig. 4 : With increase in the value of 'p', the total cycle time decreases and the total cost of the system increases

Conclusion:

In this paper we have developed a deterministic production inventory model for deteriorating items with time dependent deterioration and selling price dependent demand when holding cost varies with time. Price discount is offered for partial deteriorated items which makes realistic demand.

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