

Probability distribution functions of weekly reference crop evapotranspiration for Solapur district of Maharashtra

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■ **ABSTRACT** : The study was carried out to estimate a reference crop evapotranspiration (ETr) at different probability level for Solapur (North latitude 17°10" to 18°32" and East longitude 74°42" to 76°15") at 483.5 msl in the semi-arid zone of Maharashtra. Weekly reference evapotranspiration (ETr) values for the period (1977-2007) were computed by best method Penman-Monteith FAO-56. The probability distributions that were fitted to ETr values are log normal, Gumbel and Weibulls probability distribution functions. Chi-square test was perform to know the probability distribution of best fit. ETr values at 10 per cent to 90 per cent probability level for Penman-Monteith method using probability distribution of best fit.

■ **KEY WORDS** : Evapotranspiration, Probability distribution

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The crop evapotranspiration usually determined by estimating references crop evapotranspiration (ETr) and crop co-efficient (kc). Knowledge of evapotranspiration is essential for efficient management of water resource, crop production and environment assessment. It further continues to be foremost important in water resource planning and management.

Probability distribution function :

The irrigation planning should be based on the probabilistic approach and for this purpose it is necessary to know the ETr values at the different probability levels. Therefore, it is essential to know the probability distribution of ETr. In addition to this, probability analysis can be used for prediction of occurrence of future events from available records. Therefore, in this study it is planned to fit the three probability distribution functions to ETr data. The probability distributions that are used for evapotranspiration data are: Normal distribution (Ingle,1993), Log normal distribution (Dalvi and Thakur, 1990) and Gamma distribution (Rajkumar and Kumar, 2007; Kulshrestha *et al.*, 2007 and Kumar *et al.*, 2007). They used Chi-square test (Dalvi and Thakur, 1990 and Suresh, 2003) and Kolmogorov–Smirnov test (Kulshrestha *et al.*, 2007) to test the goodness of fit of parametric probabilistic distribution to the given set of data. In this study, therefore,

log normal, Gumbel and Weibull's probability distribution functions were proposed to fit to reference crop evapotranspiration. The Chi-square test of goodness of fit was used to select best fit distribution.

■ METHODOLOGY

The study was carried out to estimate a reference crop evapotranspiration (ETr) at different probability level for Solapur.

Collection of historical rainfall data :

In this study, meteorological data: Daily parameters (*i.e.* maximum temperature (T_{max} , °C) and minimum temperature (T_{min} , °C), maximum relative humidity (RH_{max} , %) and minimum relative humidity (RH_{min} , %), pan evaporation (E_{pan} , mm), wind speed (WS, kmhr⁻¹) at height of 2.0 m, sun shine hours (SSHr, hr), rainfall (R, mm) etc. collected from Indian meteorological Department, Pune.

Estimation of reference crop evapotranspiration (E_r) :

The weekly reference crop evapotranspiration estimated by using the standard method *i.e.* Penman- Monteith (Allen *et al.*, 1998) for the present study and is given by :

$$ETr \ N \frac{0.408 (R_n > G) < \frac{900}{T < 273} u_2 (e_s > e_a)}{< (1 < 0.34 u_2)} \dots\dots(1)$$

where,

ETr = reference evapotranspiration (mm/day),

G = soil heat flux density (MJ/m²/day),

R_n = net radiation (MJ/m²/day),

T = mean daily air temperature (°C),

= psychrometric constant (kPa/°C),

Δ = Slope of saturation vapour pressure function (kPa/°C),

e_s = Saturation vapour pressure at air temperature T (kPa),

e_a = Actual vapour pressure at dew point temperature (kPa),

u₂ = average daily wind speed at 2 m height (m/sec).

Method of probability distribution analysis :

Gumbel distribution :

The density function of Gumbel (extreme value type-I) for maxima distribution to fit an observed data can be expressed as:

$$f(x) \ N \frac{e^{-\frac{(x-y)}{s}} e^{-\frac{(x-y)}{s}}}{s} \dots\dots(2)$$

where,

X = variate of the sample

β = scale parameter

γ = location parameter.

Weibull (Maxima) distribution :

The density function of Weibull (extreme value type-III) for maxima distribution to fit an observed data can be expressed as:

$$f(x) \ N \frac{\gamma}{s} \frac{x > x}{s} e^{-\frac{x > x}{s} \gamma} \dots\dots(3)$$

where,

α = shape parameter

β = scale parameter

γ = location parameter

X = variable of the sample.

Log normal distribution :

This is transformed normal distribution in which the variable is replaced by logarithmic value. The equation is expressed as :

$$f(x) \ N \frac{1}{x \sqrt{2\pi} \sigma} e^{-\frac{(\ln x - \mu)^2}{2\sigma^2}} \dots\dots(4)$$

where,

X = variable of the sample

μ = population mean

σ = standard deviation.

Test for goodness of fit of probability distributions :

In this study, Chi-square test selected among the most commonly useful procedures for testing goodness of fit test. The test statistic (c²) will be estimated from the expression :

$$x^2 \ N \ \sum_{i=1}^k \frac{(O_i > E_i)^2}{E_i}$$

Where,

k = number of years

O_i = observed values in ith year

E_i = expected value in ith year.

Selection of best fit distribution :

For each week different probability distribution functions were tested using Chi-square test. In this calculated Chi-square value was compared with table value and observed the significance at 5 per cent level of significance. If the calculated Chi-square value was found less than the table value, that distribution for respective week was considered as non significant. If more than one distribution was found fit for all weeks or months, then distribution with lowest Chi-square value were selected as best fit distribution for respective week or month. Out of these best fit distribution, the distribution best fit for more number of weeks was selected to determine the expected reference evapotranspiration rate at 10, 20, 30, 40, 50, 60, 70, 80, and 90 per cent probability levels for each week and month.

The data were analyzed by VTFIT software. VTFIT happens to be a routine for fitting homogeneous probability density functions (PDFs) which fits PDFs to data by maximum likelihood method. The VTFIT package used for fitting probability functions that provide goodness of fit tests and expected values.

RESULTS AND DISCUSSION

The results obtained from the present investigation as well as relevant discussion have been summarized under following heads :

Probability distribution function for ETr :

From the Table 1 it is observed that none of the probability distribution fit to all weeks. All the probability distributions under consideration fit to week no.1, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 45, 47, 48, 49, 50, 51 and 52. None of the probability distribution fit to week no.3. It is observed from the Table 1 that log normal distribution function fits to maximum number of weeks (49) Gumbel distribution (total week 46) and Weibulls distribution (total week 45). As more than one distribution fits to many weeks, the best distribution for such week was considered as the distribution that gives the lowest values of Chi-square at 5 per cent level of significance. The log

Table 1 : Number of weeks under various probability distribution functions fitting to weekly values of ETr obtained by using Penman-Monteith method for Solapur district

Types of distributions	Week numbers	Total (week)
Log normal	1, 2,4,5,6,8,9,10,11,12,13,14,15,16,17, 18,19,20,21,22,23,24,25,26,27,28,30,31 32,33,34,35,36,37,38,39,40,41,42,43, 44,45,46,47,48,49,50,51,52	49
Gumbel	1, 4,6,8,10,11,12,13,14,15,16,17, 18,19,20,21,22,23,24,25,26,27,28,29,30, 31,32,33,34,35,36,37,38,39,40,41,42,43, 44,45,47,48,49,50,51,52	46
Weibull	1,2,7,8,9,10,11,12,13,14,15,16,17, 18,19,20,21,22,23,24,25,26,28,29,30, 32,33,34,35,36,37,38,39,40,41,42,43, 45,46,47,48,49,50,51,52	45

normal distribution is the best fit for maximum weeks (34), followed by Gumbel (12), and Weibull's (5).

Estimation of ETr values at different probability levels :

ETr values at different probability levels viz., 10, 20, 30, 40, 50, 60, 70, 80 and 90 per cent were estimated by using these best probability distribution functions for all the weeks. In case of more than one distribution is the best fit, the distribution selected for estimating ETr values was in order of log normal, Gumbel and Weibull's (Meshram, 2010). The values of ETr by Penman-Monteith method at different probability levels for Solapur for all the weeks are shown in Fig 1.

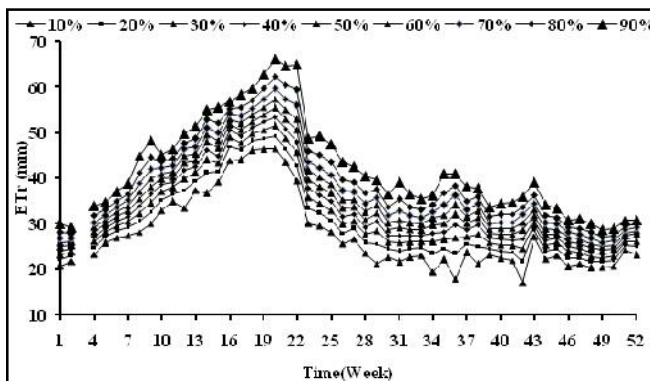


Fig. 1 : Weekly ETr values at different probability levels by using best fit probability distribution functions for Penman-Monteith method for Solapur district

Conclusion :

The FAO Penman-Monteith method was used as a standard. The mean annual ETr value is 1803.7 mm, which varies from 1560.58 to 2070.82 mm with a CV of 7.60 per cent. The annual ETr series has a co-efficient of skewness of 0.05 and a kurtosis of -0.98. log normal distribution is the best fit for maximum weeks (34), followed by Gumbel (12), and Weibull's (6) for Solapur district.

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