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A muskingum model based on unit-step and transfer function approach for prediction of direct runoff hydrographs from a small watershed

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ABSTRACT : The hydrological investigation was carried out to develop a mathematical expression for Muskingum model on the basis of application of unit-step function for prediction of direct runoff hydrographs from Shenda Park watershed, Kolhapur of Maharashtra state considering it to be a lumped, linear and time-invariant system. Generally the ordinates of direct runoff are obtained directly as the inverse Laplace transform of the product of Laplace transform of the input and the transfer function of the system. The value of model parameter, storage constant (K) was estimated, which was found to be 0.37 (hr). Direct runoff hydrographs were developed against three values of weighing factor, X=0.00 (reservoir routing), X=0.05 (channel routing), and X=0.10 (channel routing). Performance evaluation of developed model in determining direct runoff hydrograph ordinates was evaluated using various statistical indices. For weighing factor, X=0.00, the overall average values of co-efficient of efficiency (CE), co-efficient of correlation (R), special correlation co-efficient (R_s), root mean square error (RSME) and percentage absolute deviation in peak flow (PAD_p) and runoff volume (PAD_v) were found to be 0.902, 0.962, 0.926, 0.0013 and 17.66 and 2.65, respectively. Based on all the evaluation criteria, model can be easily applied for the prediction of direct runoff hydrograph ordinates for the study watershed.

KEY WORDS : Direct runoff hydrograph, Muskingum model, Laplace transform

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The problem of estimating runoff from small watersheds is an important element in the design of soil and water conservation structures, such as spillways, check dams, diversion works, bunds, contour trenches, water harvesting ponds. Estimation of flood hydrographs can be achieved by different methods and hydrological models are one of them. The transformation of rainfall to runoff is a complex physical phenomenon, which is yet to be fully understood. In the hydrology of natural catchments, rainfall-runoff relations are usually non-linear. However, the linear theory is frequently adopted because it is mathematically much easier to handle than the better fitting non linear models. Therefore, assumption of linearity and time-invariant has been considered a convenient starting point for handling input-output relationships in hydrologic study. Ideally, a conceptual model based on sound physical principles would be the best approach to predict runoff from a given rainfall. Among the many hydrologic models used for flood routing in natural channels and reservoirs, the Muskingum model has been one of the most frequently used tools, because of its simplicity and involvement of fewer parameters.

While applying linear conceptual models, generally the parameters of the impulse response function or the instantaneous unit hydrograph (IUH) are obtained from the effective rainfall and direct runoff data. The IUH is converted into a unit hydrograph of finite duration. The outflow hydrograph is obtained by the convolution of the effective rainfall with the unit hydrograph. The parameters of IUH are obtained as in the general procedure. The outflow can be obtained by taking inverse Laplace transform of the product of the Laplace transform of the instantaneous unit hydrograph and the input *i.e.* effective rainfall. In the present study, Muskingum model using unit-step and transfer function approach as suggested by Wang and Wu (1983) was developed for determining direct runoff hydrographs for small watershed of 12 ha area developed at National Agricultural Research Project (NARP), Shenda Park, Kolhapur of Maharashtra state, India, considering watershed as a lumped, linear and time-invariant system.

■ METHODOLOGY

A small watershed comprising an area of 12 ha and has leaf shape developed at NARP, Shenda Park, Kolhapur (Maharashtra) was selected for this study. The study area is located at 16°45' N latitude and 74°14' E longitude. The altitude of the watershed is 574.0 m above mean sea level. The climate is characterized by relatively hot summer, humid and cool rainy season and moderately cold winter. The mean annual rainfall of the study area is 1040 mm in 68 rainy days. About 93 per cent rainfall is received through South-West monsoon during June to September. Maximum rain falls in July and August and the watershed falls under sub-humid regions of Maharashtra. The rainfall and runoff data of Shenda Park watershed was collected for the years 2000 to 2008 from zonal station of National Agricultural Research Project, Shenda Park, Kolhapur (M.S.). In this study, twelve single peaked and isolated storm events were selected. A calibration set containing nine events and verification sets consist of three storm events which were used for estimating model parameters and for prediction purpose.

Input-output relationship for muskingum model :

Application of system analysis in hydrology has brought about one of the greatest advances in modern hydrological technology. Generally speaking, a system consists of an input, an output and the transformation whereby the input is transformed into the output. In the hydrological context, a basin is considered as the system in which an input of effective rainfall is transformed in to an output as discharge at the outlet. In the analysis or study of a system an appropriate model must be selected. In this study, Muskingum model using unit-step and transfer function approach as suggested by Wang and Wu (1983) was developed for determining direct runoff hydrographs from the study watershed. The input output relationship of a linear time-invariant system of a basin can be represented by the linear differential equation reported by (Ogata, 1970) is given as :

$$a_n \frac{d^n Q(t)}{dt^n} + a_{n-1} \frac{d^{n-1} Q(t)}{dt^{n-1}} + \ldots,\ldots + a_1 \frac{Q(t)}{dt} + a_0 Q(t) = b_m \frac{d^m I(t)}{dt^m} + b_{m-1} \frac{d^{m-1} I(t)}{dt^{m-1}} + \ldots + b_1 \frac{I(t)}{dt} + b_0 I(t) \quad .....(1)$$

In which, Q and I are the output (*i.e.* the outflow) and the input (*i.e.* the effective rainfall), respectively and $a_n, a_{n-1}, \ldots a_1, a_0$; $b_m, b_{m-1}, \ldots.. b_1, b_0$ are the positive integers with $n > m$ (Kulandaiswamy and Babu, 1975). The detailed analytical derivation of the model has been described by Kumar *et al.* (2008).

Derivation of outflow for muskingum model :

The equation of continuity used in all hydrologic routing as the primary equation, states that the difference between the inflow and outflow rate is equal to rate of change of storage and it is expressed as :

$$\frac{ds}{dt} = I - Q \quad .....(2)$$

where, I is the inflow rate, Q is the outflow rate and S is the storage.

The linear storage-discharge relation (*i.e.* S in terms of I and Q) known as the Muskingum equation, can be written as:

$$S = K\,[X\,I + (1 - X)\,Q\,] \quad .....(3)$$

In which, K is storage time constant and has dimension of time and X is the dimensionless co-efficient used to weigh the relative effects of inflow and outflow on reach storage and is known as weighing factor. If the X is zero, the inflow values have no bearing on the storage capacity in the reach as in case of reservoir type storage. When X is equal to 0.5 both inflow and outflow have equal weight and there is no attenuation in peak. In the present study, direct runoff hydrographs were derived for three values of X such as X=0.00 (reservoir routing), X=0.05 (channel routing) and X=0.10 (channel routing).

Differentiation of equation (3) with respect to time yields:

$$\frac{ds}{dt} = KX\frac{dI}{dt} + K(1 - X)\frac{dQ}{dt} \quad .....(4)$$

Equation (2) and (4) results in the differential form of the Muskingum model, expressed by the equation :

$$K(1 - X)\frac{dQ}{dt} + Q = 1 - KX\frac{dI}{dt} \quad .....(5)$$

Taking Laplace transform of both the sides of equation (5) and on simplification, the resulting equation is expressed as :

$$\frac{Q(s)}{I(S)} = \frac{1 - KXs}{K(1 - X)s + 1} \quad ......(6)$$

From the equation (6) and input-output relationship as suggested by Ogata (1970), transfer function of the system is given as :

$$H(s) = \frac{1}{K(1 - X)^2}\,\frac{1}{\left[s + \frac{1}{K(1 - X)}\right]} - \frac{x}{1 - x} \quad .....(7)$$

In this paper the rainfall data in "blocks" of finite duration is represented by the unit-step function. If m consecutive effective rainfall amounts, expressible as P_1, P_2, P_3, ….., Pm each occurring for a time interval Δt, the input in terms of the unit-step function can be expressed as :

$$I(t) = p_1 u(t) + (p_2 - p_1)\, u(t-\Delta t) + (p_3 - p_2)\, u(t-2\Delta t) + \ldots + (p_m - p_{m-1})\, u[t-(m-1)\Delta t] - p_m\, u(t-m\Delta t) \quad \ldots(8)$$

Further simplifying the resulted equations, final equation for derivation of direct runoff hydrographs in the form of summation is expressed as :

$$Q(t) = \sum_{i=0}^{m} (P_{i+1} - P_i)\left[1 - \frac{1}{(1-X)} e^{-\frac{(t-i\Delta t)}{K(1-X)}}\right] u(t - i\Delta t) : P_0 = 0,\ P_{m+1} = 0$$

where Q(t) is the ordinate of the direct runoff hydrograph at time t, P_i is i[th] effective rainfall, Δt is time interval and m is total number of rainfall blocks.

Estimation of model parameter :

In order to determine the outflow hydrograph using equation (8), the parameter to be estimated is storage time constant, K. the value of K can be determined by using method suggested by Jawed (1973). According to him the value of storage constant K is determined considering the discharge at the time of the maximum slope on the recession of the semi-log hydrograph of the recession curve based relationship :

$$K = -\frac{Q_i}{\frac{\Delta Q}{\Delta T}} \quad (10)$$

where Q_i is the discharge at point of inflection, $\Delta Q/\Delta T$ is the slope of the straight line passing through point of inflection, and ΔQ is the incremental runoff rate for incremental time ΔT. The estimated values of storage time constant for nine storm event (calibrated event) are given in the Table A.

By substituting the average value of storage time constant (K) = 0.37 (hr) in the equation (8), the final expression for Muskingum model for prediction of direct runoff hydrographs from study watershed is given as :

$$Q(t) = \sum_{i=0}^{m} (P_{i+1} - P_i)\left[1 - \frac{1}{(1-X)} e^{-\frac{(t-i\Delta t)}{0.37(1-X)}}\right] u(t - i\Delta t) : P_0 = 0,\ P_{m+1} = 0$$

Performance evaluation of model :

To evaluate the model, five statistical parameters *viz.*, correlation co-efficient (R) (Sarma *et al.*, 1973), special correlation co-efficient (R) (Eagleson and March, 1965), co-efficient of efficiency (CE) (Nash and Sutcliffe, 1970), root mean square error (RMSE) (Yu *et al.*, 1994) and the percentage absolute deviation in peak flow rates (PAD_p) and percentage absolute deviation in direct runoff volumes (PAD_v) (Wang *et al.*, 1992) were used for the purpose :

$$R = \frac{\left[N \sum_{i=1}^{N} (Q_{oi}\, Q_{ci}) - \left(\sum_{i=1}^{N} Q_{oi}\right)\left(\sum_{i=1}^{N} Q_{ci}\right)\right]}{\sqrt{\left[N \sum_{i=1}^{N} (Q_{oi})^2 - \left(\sum_{i=1}^{N} Q_{oi}\right)^2\right] \times \left[N \sum_{i=1}^{N} (Q_{ci})^2 - \left(\sum_{i=1}^{N} Q_{ci}\right)^2\right]}} \quad ..(12)$$

$$R_S = \left[\frac{2\sum_{i=1}^{N} Q_{oi}.Q_{ci} - \sum_{i=1}^{N} (Q_{ci})^2}{\sum_{i=1}^{N} (Q_{oi})^2}\right] \quad \ldots\ldots(13)$$

$$CE = \frac{\sum_{i=1}^{N} (Q_{oi} - \overline{Q}_o)^2 - \sum_{i=1}^{N} (Q_{oi} - Q_{ci})^2}{\sum_{i=1}^{N} (Q_{oi} - \overline{Q}_o)^2} \quad \ldots\ldots(14)$$

$$RMSE = \sqrt{\frac{1}{N} \sum_{1=1}^{N} (Q_{oi} - Q_{ci})^2} \quad \ldots\ldots(15)$$

$$PAD_p = \frac{|Q_{po} - Q_{pc}|}{Q_{po}} \times 100 \quad \ldots\ldots.(16)$$

$$PAD_T = \frac{|T_{po} - T_{pc}|}{T_{po}} \times 100 \quad \ldots\ldots(17)$$

Table A : Estimated values of storage constant (K) for calibrated storm events

Storm event	Direct runoff rate at point of inflection (Q_i) m^3/s	Incremental runoff rate (ΔQ) m^3/s	Incremental time (ΔT) hr	Storage constant K (hr)
July 12, 2000	0.035	0.027	0.30	0.46
October 9, 2000	0.045	0.036	0.25	0.42
June 29, 2005	0.150	0.120	0.50	0.30
July 4, 2005	0.019	0.012	0.20	0.32
July 25, 2005	1.500	1.490	0.55	0.39
August 9, 2005	0.090	0.080	0.30	0.34
September 8, 2005	0.080	0.071	0.29	0.38
September 23, 2005	0.030	0.020	0.31	0.36
July 29, 2006	0.090	0.080	0.45	0.39
Average value				0.37

where, Q_{oi} is the observed storm runoff hydrograph ordinates at i^{th} time; Q_{ci} is the computed storm runoff hydrograph ordinates at i^{th} time; N is total number of ordinates; $\overline{Q}_o$ is mean of observed storm runoff hydrograph ordinates; Q_{po} is the observed peak flow rates; Q_{pc} is computed peak flow rates; V_o is observed direct runoff volume and V_c is computed direct runoff volume.

RESULTS AND DISCUSSION

From total of twelve storm events, nine storm events were used to calibrate the model while three storm events were used for the validation purpose of the model. The performance of the model was tested for three different values of weighing factor X, *viz.*, X=0.00 (reservoir routing), X=0.05 (channel routing), and X=0.10 (channel routing) by comparing observed and predicted direct runoff hydrographs for the events of September 8, 2005 and June 15, 2004 one event each of the calibration and verification sets as shown in (Fig. 1 and 2) and for both the cases good approximations to the actual runoff hydrographs are noted.

From the figures, it is clear that rising, crest, and recession segments of computed hydrographs are in close agreement with those of observed direct runoff hydrographs whereas there is increasing tread in peak flow values for the values of X=0.00, X=0.05 and X= 0.10, respectively.

The estimated values of all statistical indices *viz.*, correlation co-efficient (R), special correlation co-efficient (R),

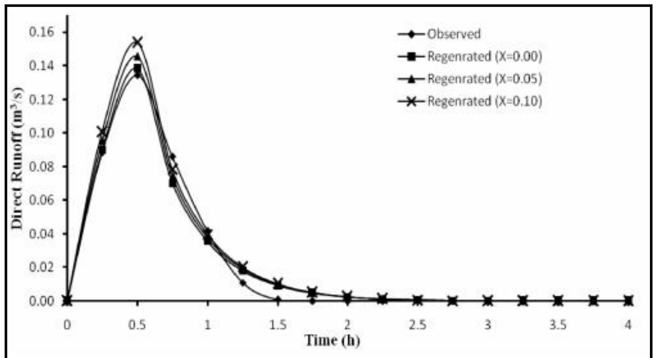

Fig. 1 : Observed and regenerated direct runoff hydrographs for the storm event of September 8, 2005

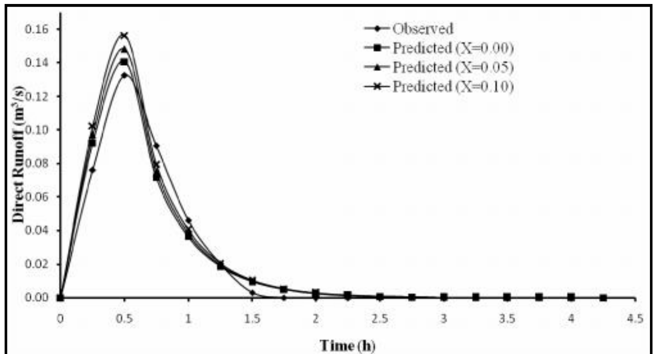

Fig. 2 : Observed and predicted direct runoff hydrographs for the storm event of June 15, 2004

Table 1 : Statistical performance evaluation indices, co-efficient of efficiency (CE), correlation co-efficient (R) and special correlation co-efficient (R_s)

Storm event	CE			R			R_s		
	X=0.00	X=0.05	X=0.10	X=0.00	X=0.05	X=0.10	X=0.00	X=0.05	X=0.10
July 11-12, 2000	0.971	1.000	0.942	0.991	0.992	0.991	0.979	0.971	0.944
October 9, 2000	0.994	0.988	0.974	0.997	0.997	0.997	0.995	0.991	0.980
June 29, 2005	0.987	0.885	0.826	0.994	0.994	0.994	0.989	0.910	0.864
July 4, 2005	0.686	0.671	0.652	0.831	0.813	0.831	0.736	0.724	0.708
July 25, 2005	0.945	0.923	0.889	0.982	0.982	0.982	0.958	0.942	0.916
August 9, 2005	0.942	0.926	0.899	0.975	0.975	0.975	0.955	0.943	0.922
September 8, 2005	0.984	0.982	0.972	0.992	0.992	0.992	0.987	0.986	0.978
September 23, 2005	0.898	0.869	0.826	0.966	0.966	0.966	0.926	0.905	0.873
July 29, 2006	0.835	0.792	0.732	0.955	0.955	0.955	0.881	0.851	0.808
Average value	0.916	0.893	0.857	0.965	0.963	0.965	0.934	0.914	0.889
*June 15, 2004	0.968	0.960	0.943	0.983	0.981	0.978	0.975	0.969	0.956
*July 2, 2006	0.820	0.777	0.717	0.950	0.949	0.947	0.873	0.843	0.800
*August 9-10, 2008	0.875	0.852	0.871	0.945	0.950	0.947	0.906	0.889	0.863
Average value	0.888	0.863	0.844	0.959	0.960	0.957	0.918	0.900	0.873
Total Average value	0.902	0.878	0.850	0.962	0.962	0.961	0.926	0.907	0.881

* Predicted storm events

co-efficient of efficiency (CE), root mean square error (RMSE), and the percentage absolute deviation in peak flow rates (PAD_p) and percentage absolute deviation in direct runoff volumes (PAD_v)are presented in the Table 1 and 2.

It is evident from Table 1, the average values of co-efficient of efficiency (CE) for regenerated storm events were found to be 0.916, 0.893 and 0.857 while for predicted storm events, it is found to be 0.888, 0.863 and 0.884 for the values of X=0.00, X=0.05 and X=0.10, respectively. The total average values of co-efficient of efficiency were found to 0.902, 0.878 and 0.850 for the values of X=0.00, X=0.05 and X=0.10, respectively. Chiew *et al*. (1993) classified the co-efficient of efficiency into three categories *viz.*, perfectly acceptable simulation (CE > 0.90), acceptable simulation (0.60 < CE < 0.90) and unacceptable simulation (CE < 0.60). On the basis of above classification criterion, developed model comes under perfectly acceptable simulation category for X=0.00 and in acceptable simulation category for the value of X=0.05 and X=0.10.

The average values of co-efficient of correlation (R) for regenerated storm events were found to be 0.965, 0.963 and 0.965 while for predicted storm events, it is found to be 0.959, 0.960 and 0.957 for the values of X=0.00, X=0.05 and X=0.10, respectively. The overall average values of co-efficient of correlation (R) were found to be 0.962, 0.962 and 0.961 for the values of X=0.00, X=0.05 and X=0.10, respectively. Sarma *et al*. (1973) reported the ratings of the statistical measures for correlation co-efficient (R) as: 0.99 =R< 1.0 excellent, 0.95 = R < 0.99 very good, 90 = R <0.95 good, 0.85 = R < 0.90 fair and 0.00 = R < 0.85 poor. Based on the above ratings, the developed model falls under very good category for three value of X=0.00, X=0.05 and X=0.01.

The average values of special correlation co-efficient (R_s) for regenerated storm events were found to be 0.934, 0.914 and 0.899 while for predicted storm events, it is found to be 0.918, 0.900 and 0.889 for the values of X=0.00, X=0.05 and X=0.10, respectively. The overall average values of co-efficient of correlation (R_s) were found to be 0.926, 0.907 and 0.881 for the values of X=0.00, X=0.05 and X=0.10, respectively. As per the rating reported by Sarma *et al*. (1973) the developed model falls under good category for values of X=0.00 and X=0.05, while for X=0.10 it falls under fair category.

It can be seen from Table 2, that The average values of percentage absolute deviation in peak flow rate (PAD_p) for regenerated storm events were determined to be 18.26, 24.38 and 30.58 while for predicted storm events, it is found to be 17.06, 23.22 and 30.07 for the values of X=0.00, X=0.05 and X=0.10, respectively. The overall average values of absolute deviation in peak flow rate were found to be 17.66, 23.80 and 30.32 for the values of X=0.00, X=0.05 and X=0.10, respectively. The average values of percentage absolute deviation in total runoff volume (PAD_v) for regenerated storm events were determined to be 2.64, 11.81 and 18.20 while for predicted storm events, it is found to be 2.67, 8.07 and 14.08 for the values of X=0.00, X=0.05 and X=0.10, respectively. The overall average values of percentage absolute deviation in total runoff volume were found to be 2.65, 9.94 and 16.14 for the values of X=0.00, X=0.05 and X=0.10, respectively. The low values of percentage absolute deviation peak flow rate and in total runoff volume for developed model shows good prediction for X=0.00 and X=0.05 while in acceptable range for the X=0.10.

From Table 2, it is seen that, the average values of root

Table 2 : Statistical performance evaluation indices, percentage absolute deviation (PAD) and root mean square error (RSME)

Storm event	PAD						RMSE		
	X=0.00		X=0.05		X=0.10		X=0.00	X=0.05	X=0.10
	PAD_v	PAD_p	PAD_v	PAD_p	PAD_v	PAD_p			
July 11-12, 2000	2.51	14.16	6.60	20.17	14.07	26.84	0.0007	0.0008	0.0011
October 9, 2000	2.69	4.80	8.09	10.32	14.10	16.44	0.0005	0.0007	0.0010
June 29, 2005	2.67	7.39	27.54	33.41	34.63	40.82	0.0009	0.0025	0.0031
July 4, 2005	2.66	29.30	23.62	14.85	30.51	14.85	0.0016	0.0016	0.0017
July 25, 2005	2.68	20.39	8.08	26.73	14.10	33.77	0.0028	0.0033	0.0039
August 9, 2005	2.69	15.98	8.09	22.08	14.10	28.86	0.0021	0.0024	0.0028
September 8, 2005	2.52	2.90	8.09	8.31	14.10	14.33	0.0012	0.0012	0.0016
September 23, 2005	2.69	29.04	8.09	35.83	14.10	43.38	0.0008	0.0009	0.0010
July 29, 2006	2.67	40.37	8.09	47.76	14.10	55.97	0.0035	0.0039	0.0044
Average value	2.64	18.26	11.81	24.38	18.20	30.58	0.0016	0.0019	0.0023
*June 15, 2004	2.68	6.16	8.08	11.74	14.09	17.95	0.0016	0.0018	0.0022
*July 2, 2006	2.63	31.38	8.03	38.30	14.03	45.98	0.0013	0.0015	0.0016
*August 9-10, 2008	2.69	13.64	8.09	19.62	14.10	26.27	0.0001	0.0012	0.0013
Average value	2.67	17.06	8.07	23.22	14.08	30.07	0.0010	0.0015	0.0017
Total average value	2.65	17.66	9.94	23.80	16.14	30.32	0.0013	0.0017	0.0020

* Predicted storm events

mean square error for regenerated storm events were estimated 0.0016, 0.0019 and 0.0023 for X=0.00, X=0.05 and X=0.10, respectively while for predicted storm events these were found to be 0.0010, 0.0015 and 0.117 for X=0.00, X=0.05 and X=0.10, respectively. The overall average values of root mean square error for developed model were determined to be 0.0013, 0.0017 and 0.0020 for X=0.00, X=0.05 and X=0.10, respectively, which are nearly equal to zero, hence, the performance of the developed model in predicting direct runoff hydrograph from study watershed is satisfactory.

Conclusion :

Closer agreement between the rising segment, crest segment, recession segment of regenerated and observed direct runoff hydrographs; lower values of percentage absolute deviation in direct runoff rate and peak flow rate and root mean square error and maximum values of co-efficient of efficiency, co-efficient of correlation and special correlation co-efficient, for X=0.00 (reservoir routing) and X=0.05 (channel routing) showed high degree of goodness of fit which indicates that developed Muskingum model based on unit step and transfer function can be applied to predict direct runoff hydrographs from the watershed developed at National Agricultural Research Project (NARP), Shenda Park, Kolhapur of Maharashtra state.

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