Asian Science
Volume 7 | Issue 1 | June, 2012 | 90-106

RESEARCH PAPER

Food web cycle-green plants and nutrients concatenated to terrestial organism-oxygen consumption-decomposer organism-dead organic matter-accentuation-A trophication model

K.N. PRASANNA KUMAR*, B.S. KIRANAGI AND C.S. BAGEWADI
Department of Mathematics, Kuvempu University, SHIMOGA (KARNATAKA) INDIA

ABSTRACT

A system of green plants absorbing nutrients *vis-à-vis* decomposer organisms attested to terrestrial organisms dissipating consumption of oxygen due to cellular respiration and parallel system of consumption of dead organic matter concatenated to oxygen due to cellular respiration that contribute to the dissipation of the velocity of production of decomposer organisms *vis-à-vis* terrestrial organisms is investigated. It is shown that the time independence of the contributions portrays another system by itself and constitutes the equilibrium solution of the original time independent system. A system of nutrients consolidated with dead organic matter that reduces the dissipation coefficient of the green plants correlated to decomposer organism annexed to the oxygen consumption-terrestrial organism system. With the methodology reinforced and revitalized with the explanations, we write the governing equations with the nomenclature for the systems in the foregoing. Further papers extensively draw inferences upon such concatenation process thus consummating the fait accompli desideratum of the food web cycle, towards which the consubstantiation process was undertaken for execution.

Key Words : Food web cycle-green plants, Organism-oxygen, Organism-dead organic matter

View point paper : Prasanna Kumar, K.N., Kiranagi, B.S. and Bagewadi, C.S. (2012). Food web cycle-green plants and nutrients concatenated to terrestial organism-oxygen consumption-decomposer organism-dead organic matter-accentuation-A trophication model. *Asian Sci.*, **7**(1): 90-106.

In his celebrated paper Haimovici (1982), studied the growth of a two species ecological system divided on age groups. In this paper, we establish that his processual regularities and procedural formalities can be applied for consummation of system of oxygen consumption by terrestrial organisms. Notations are changed towards the end of obtaining higher number of equations in the holistic study of the global climate models. Quintessentially, Haimovician diurnal dynamics, are used to draw interesting inferences, from the simple fact that terrestrial organisms consume oxygen due to cellular respiration.

Capra in his scintillating and brilliant synthesis of such scientific breakthroughs as the "Theory of Dissipative structures", 'Theory of complexity', 'Gia theory', 'Chaos theory' in his much acclaimed 'The Web of life' elucidates dissipative structures as the new paradigm in ecology.

Heylighen (2001) also concretises the necessity of self-organization and adaptability. Matsuit *et al.* (2006) made a satellite based assessment of marine low cloud variability, atmospheric stability and diurnal cycle. Steven's Feingold (2010) studied untangling aerosol effects on clouds and precipitation in a buffered system. Illan koren and Graham

*** Author for correspondence**
B.S. Kiranagi, Department of Studies in Mathematics), University of Mysore, MYSORE (KARNATAKA) INDIA
C.S. Bagewadi, Department of Studies in Mathemtics and Computer Science, Kuvempu University, Shankarghatta, SHIMOGA (KARNATAKA) INDIA

Feingold *et al.* (2010) studied the aerosol cloud precipitation system. One other study that eminently calls for such a study of application is by Wood (2007) in which he studied the loss of cloud droplets by coalescence in warm clouds. On the same lines the investigation of Xue H, Fiengold G where in indirect effects of aerosol on large eddy simulations of trade wind provides a rich repository and fertile ground for prosecution of investigation based on our theoretical analysis. Aerosol effects on clouds itself is a pointer to the food cycle—dissipative structure discussed by Prigogine.

All the studies centre on the possibility of application of Haimovician analysis to "dissipative structures". In this paper we study the following systems:

– Oxygen consumption-Terrestrial organism
– Dead organic matter-Decomposer organisms

We elucidate the governing equations of (b) Methodology for obtaining of solution follows from the one herein given

In the next part we analyze the following systems:

– Plant investment-Nutrients
– Solar radiation-Chemical process
– Systems structure-Change

Green plants play a vital role in the flow of energy through all ecological cycles. Their roots take in water and mineral salts from the earth, and the resultant juices rise up to the leaves, where they combine with CO_2 from air leading to the formulation of sugar and other organic compounds. Here solar energy is converted into chemical energy and encapsulated in organic substances, while oxygen is released in air to be taken up again by other plants and by animals in the process of cellular respiration. By the blend of water and minerals with sunlight and CO_2, green plants form link between earth and sky. Bulk of cellulose and the other organic compounds produced through photosynthesis consists of heavy carbon and oxygen atoms, which plants take directly from the air in the form of CO_2. Thus the weight of a wooden log comes almost entirely from air. A log burnt, combines oxygen and carbon combine once more in to CO_2 and in the light and heat of fire is recovered part of the solar energy that went into making the wood.

As terrestrial organisms dissipate oxygen in the atmosphere, due to cellular respiration the plants nutrients are passed through the food web, while energy is dissipated as heat through respiration and as waste through excretion. Dead animals and plants are disintegrated by decomposer organisms, which break them into basic nutrients to be taken up by plants. Nutrients and other basic elements continually cycle through the ecological system, while energy is dissipated at each stage in accord with Eugene Odum's dictum "matter circulates, energy dissipates". Waste generated by the ecological system as a whole is the heat energy of cellular respiration, which is radiated into the atmosphere and is reimbursed continually by photosynthesis.

Prigogine's theory interlinks/entangles the main characteristics of living forms in to a coherent, cogent conceptualization and mathematical framework. We give a model for his framework. Perhaps the most fundamental necessity of the systemic dynamics is the optimality considerations. Taking cognizance of the critical issues involved emphasizes need for setting out dynamic programming in order to capture systemic structural changes.

Axiomatic predications of systemic dynamics in question are essentially "laws of accentuation and dissipation'. It includes once over change, continuing change, process of change, functional relationships, predictability, cyclical growth, cyclical fluctuations, speculation theory, cobweb analyses, stagnation thesis, perspective analysis etc. Upshot of the above statement is data produce consequences and consequences produce data.

Nutrients *vis a vis* dead organic matter *vis a vis* oxygen consumption due to cellular respiration:

Assumptions :

– Nutrients(NR) reinforced with DEAD ORGANIC MATTER(DOM-) concatenated with Oxygen Consumption due to cellular respiration are classified into three categories;

- Category 1 representative of the NR-DOM CONCATENATED WITH oxygen consumption due to cellular respiration in the first interval vis a vis category1 of terrestrial organisms

- Category 2 (second interval) comprising of NR- DOM CONSOLIDATED WITH consumption due to cellular respiration corresponding to category 2 of terrestrial organisms

- Category 3 constituting NR-dead organic matter (DOM) concretised with consumption due to cellular respiration which belong to higher age than that of category 1 and category 2.This is concomitant to category 3 of terrestrial organism

In this connection, it is to be noted that there is no sacrosanct time scale as far as the above pattern of classification is concerned. Any operationally feasible scale with an eye range of consumption due to cellular respiration. Similarly, a "less than scale" for on the terrestrial organisms made out of the total oxygen consumption due to cellular respiration would be in the fitness of things, as it would be with the quantum of dead organic matter (see capra food cycle p. 174) For category 3. "Over and above" nomenclature could be used to encompass a wider category 1 can be used.

– The speed of growth of NUTRIENTS (NR) CONCOMITANT with dead organic matter(DOM) attributable and ascribable to oxygen consumption due to cellular respiration under category 1 is proportional to the total amount of oxygen consumption due to cellular respiration under category 2. In essence the accentuation coefficient in the model is representative of the constant of proportionality between consumption due to nutrients(NR) linked to dead organic matter (DOM) consubstatiated with cellular respiration under category 1 and category 2 this assumptions is made to foreclose the necessity of addition of one more variable, that would render the systemic equations unsolvable

– The dissipation in all the three categories is attributable to the following two phenomenon :

- **Aging phenomenon** : The aging process leads to transference of the balance of nutrients(NR) CORELATED WITH dead organic matter (DOM)concatenated with oxygen consumption due to cellular respiration to the next category, no sooner than the age of the terrestrial organism crosses the boundary of demarcation.

- **Depletion phenomenon** : Death of consumer *viz.,* terrestrial organism dissipates the growth speed by an equivalent extent of NUTRIENTS(NR)dead organic matter (DOM). The model is not concerned with the end uses of consumption due to cellular respiration –dissipation other than for terrestrial organisms.

Notation :

G_{20} : Quantum of NR-DOM *vis-a-vis* oxygen consumption (OC) due to cellular respiration in category 1 of terrestrial organism

G_{21}: Quantum of NR-DOM *vis-a-vis* oxygen consumption (OC)due to cellular respiration in category 2 of terrestrial organism

G_{22}: Quantum of NR- DOM *vis-a-vis* oxygen consumption (OC) due to cellular respiration in category 3 of terrestrial organism

$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$: Accentuation coefficients

$(á_{20})^{(3)}, (á_{21})^{(3)}, (á_{22})^{(3)}$: Dissipation coefficients

Formulation of the system :

In the light of the assumptions stated in the foregoing, we infer the following:-

– The growth speed in category 1 is the sum of a accentuation term $(a_{20})^{(3)}G_{21}$ and a dissipation term $-(á_{21})^{(3)}G_{20}$, the amount of dissipation taken to be proportional to the total quantum NUTRIENTS(NR) *vis-à-vis* of oxygen consumption (OC) due to cellular respiration in the concomitant category of terrestrial organisms(TO).

– The growth speed in category 2 is the sum of two parts $(a_{21})^{(3)}G_{20}$ and $-(á_{21})^{(3)}G_{21}$ the inflow from the category 1 dependent on the total amount standing in that category.

– The growth speed in category 3 is equivalent to $(a_{22})^{(3)}G_{21}$ and $-(á_{22})^{(3)}G_{22}$ dissipation ascribed only to depletion phenomenon.

Model makes allowance for the new quantum of DOM RELATIVE TO consumption due to new entrants in terrestrial organisms (TO) and deceleration in the oxygen consumption (OC) attributable and ascribable to death of terrestrial organisms (TO) LEADING to the accentuation, CORROBORATION AND AUGMENTATION OF THE DOM (Dead organic matter).

Governing equations:

The differential equations governing the above system can be written in the following form

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - (a'_{20})^{(3)}G_{20} \qquad 1$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - (a'_{21})^{(3)}G_{21} \qquad 2$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - (a'_{22})^{(3)}G_{22} \qquad 3$$

$(a_i)^{(3)} > 0$, i = 20, 21, 22 4

$(á_i)^{(3)} > 0$, i = 20, 21, 22 5

$(a_{21})^{(3)} < (á_{20})^{(3)}$ 6

$(a_{22})^{(3)} < (á_{21})^{(3)}$ 7

We can rewrite equation 1, 2 and 3 in the following form

$$\frac{dG_{20}}{(a_{20})^{(3)}G_{21} - (a'_{20})^{(3)}G_{20}} = dt \qquad 8$$

$$\frac{dG_{21}}{(a_{21})^{(3)}G_{20} - (a'_{21})^{(3)}G_{21}} = dt \qquad 9$$

Or we write a single equation as

$$\frac{dG_{20}}{(a_{20})^{(3)}G_{21} - (a'_{20})^{(3)}G_{20}} = \frac{dG_{21}}{(a_{21})^{3}G_{20} - (a'_{21})^{(3)}G_{21}} =$$

$$\frac{dG_{22}}{(a_{22})^{(3)}G_{21} - (a'_{22})^{(3)}G_{22}} = dt \qquad 10$$

The equality of the ratios in equation (10) remains unchanged in the event of multiplication of numerator and denominator by a constant factor.

For constant multiples α, β, γ all positive we can write equation (10) as

$$\frac{\alpha dG_{20}}{\alpha[(a_{20})^{(3)}G_{21} - (a'_{20})^{(3)}G_{20}]} = \frac{\beta dG21}{\beta[(a_{21})^{(3)}G_{20} - (a'_{21})^{(3)}G_{21}]} =$$

$$\frac{\gamma dG_{22}}{\gamma[(a_{22})^{(3)}G_{21} - (a'_{22})^{(3)}G_{22}]} = dt \qquad 11$$

The general solution of the consumption of oxygen due to cellular respiration system can be written in the form

$\alpha_i G_i + \beta_i G_i + \gamma_i G_i = C_i e^{\lambda_i t}$ where i = 20, 21, 22 and C_{20}, C_{21}, C_{22} are arbitrary constant coefficient.

Stability analysis :

Supposing $G_i(0) = G^0_i(0) > 0$, and denoting by λ_i the characteristics roots of the system, it easily results that

– If $(á_{20})^{(3)}(á_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} > 0$ all the components of the solution, *i.e.* all the three parts of the consumption of oxygen due to cellular respiration tend to zero, and the solution is stable with respect to the initial data.

– If $(á_{20})^{(3)} (á_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$ and $(\lambda_{21} + (á_{20})^{(3)} G^0_{20}{}^{(3)} - (a_{20})^{(3)} G^0_{21} \neq 0$, $(\lambda_{21} < 0)$, the first two components of the solution tend to infinity as $t \to \infty$, and $G_{22} \to 0$, *i.e.* The category 1 and category 2 parts grows to infinity, whereas the third part category 3 of NR-DOM RELATIVISTIC TO consumption of oxygen due to cellular respiration tends to zero.

– If $(á_{20})^{(3)} (á_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$ and $(\lambda_{21} + (á_{20})^{(3)} G^0_{20}{}^{(3)} - (a_{20})^{(3)} G^0_{21} = 0$ Then all the three parts tend to zero, but the solution is not stable *i.e.* at a small variation of the initial values of G_i, the corresponding solution tends to infinity.

Actual food cycles can be understood on a much broader canvass, in which nutrient elements appear in a variety of chemical compounds. Gaia theory has refined indications of interweaving of living and non living systems throughout the biosphere. Key to comprehension of such dissipative structures is that these systems maintain themselves in a "stable state" far from equilibrium. For instance chemical and thermal equilibrium exists when all these processes come to a halt. Organism in equilibrium is a dead organism. Living organisms, like terrestrial organisms, continually maintain themselves in a state far from equilibrium. Notwithstanding the fact, that such a maintained state is stable over a period of time, the same overall holistic structure is maintained, despite continual ongoing flow and change of components.

Prigogine realized that classical thermodynamics is not the appropriate tool to explain systems far from equilibrium, owing to the fact mathematical structure is linear. Close on the heels to equilibrium, there will be "fluxes", "vortices", however, weak nevertheless. System shall evolve towards a stationary state in which generation of "entropy" (disorder) is as small as possible. By implication, there shall be a minimization problem mathematically, around the equilibrium state. In and around this range, linear equation would explain the characteristics of the system.

On the other hand, away from "equilibrium", the "fluxes" are more emphasized. Result is increase in "entropy". When this occurs, the system no longer tends towards equilibrium. On the contrary, it may encounter instabilities that culminate into newer orders that move away from equilibrium. Thus, dissipative structures revitalize and resurrect complex forms away from equilibrium state. Ludwig VAN bertlanfly called living structures open systems to emphasize their theme and potentialities and interdependence on continual flow of energy and resources (14). All these are textual and contextual investigations are epitomized in the word "recycling" in ecology.

Prigogine's statement(15) that the locus of essence of characteristics and essence of a dissipative structure cannot be derived from the properties of its parts ,but are ramifications and consequences of 'SUPRAMOLECULAR ORGANISATION'. LINEAR EQUATIONS CAN BE ANALYSED IN TERMS OF POINT ATTRACTORS, regardless and irrespective of the initial conditions of the system and it shall be attracted towards the stationery state of minimum entropy as close to equilibrium. cytoplasm, nucleolus, ribosome, gogylapparatus, lysosome, mitochondrion, adenosine and chloroplast are the parts of the plant cell (CAPRA(2).

IN PRINCIPLE, THE MODEL REPRESENTS THE FULLER COMPLEXITY OF THE SYSTEM. Timescale parameters are coupled with other variables. Towards that end, the study explores underlying simplicity of and insight in to structurally orientational and systemically canonical ideas. Process orientation has also eneged our attention in the factotum principle that plant cell is studied in further papers (see references).

From the above stability analysis we infer the following:

– The adjustment process is stable in the sense that the system of oxygen consumption converges to equilibrium.

– The approach to equilibrium is a steady one, and there exists progressively diminishing oscillations around the equilibrium point.

– Conditions 1 and 2 are independent of the size and direction of initial disturbance

– The actual shape of the time path of oxygen consumption in the atmosphere by the terrestrial organism is determined by λ, the strength of the response of the portfolio in question, and the initial disturbance

– Result 3 warns us that we need to make an exhaustive study of the behaviour of any case in which generalization derived from the model do not hold

– Growth studies as the one in the extant context are related to the systemic growth paths with full employment of resources that are available in question, in the present case terrestrial organisms–oxygen consumption-dead organic matter available for decomposer organisms

– Some authors Nober F J, Agee, Winfree were interested in such questions, whether growing system could produce full employment of all factors, whether or not there was a full employment natural rate growth path and perpetual oscillations around it. It is to be noted some systems pose extremely difficult stability problems. As an instance, one can quote example of pockets of open cells and drizzle in complex networks in marine strato cumulus. Other examples are clustering and synchronization of lightning flashes adjunct to thunderstorms, coupled studies of microphysics and aqueous chemistry.

Green plants (GP) - Decomposer organism (DO) concatenated with terrestial organism (TO) portfolio : Governing equations thereof:

Assumptions:

– GP-DOM *vis-a-vis* terrestrial organisms (TO) are classified into three categories analogous to the stratification that was resorted to in consumption of oxygen due to DOM related to cellular respiration sector. When consumption of oxygen due to cellular respiration in a particular category is transferred to the next sector, (such transference is attributed to the aging process of terrestrial organisms), terrestrial organisms(TO) from that category apparently would have become qualified for classification in the corresponding category, because we are in fact classifying terrestrial organisms(TO)-DOM –GP as consistent with that based on stratification of consumption of oxygen due to cellular respiration.

– Category 1 is representative of GP-decomposer organisms(DO) RELATIVISTIC TO terrestrial organisms(TO) corresponding to oxygen consumptions due to cellular respiration under category 1

– Category 2 constitutes those GP- DO VIS A VIS terrestrial organisms (TO) whose age is higher than that specified under the head category 1 and is in correspondence with the similar classification of oxygen consumption (OC) due to cellular respiration.

– Category 3 of terrestrial organisms encompasses those terrestrial organisms with respect to category 3 of oxygen Consumption due to cellular respiration of terrestrial organisms with respect to concomitant categorical constitution. OF green plants (GP).

It is assumed for the sake of simplicity that amount of oxygen taken in water is slowly divided into that of utilization due to terrestrial organisms, Cellular respiration, clouds, and decomposer organisms (DO) GREEN plants (GP) etc.

– The speed of growth of (GP)-terrestrial organism TO) sector in category 1 is a linear function of the amount of terrestrial organism (TO) sector in category 2 at the time of reckoning. As before the accentuation coefficient that characterizes the speed of growth in category 1 is the proportionality factor between balance in category 1 and category 2.

– The dissipation coefficient in the growth model is attributable to two factors;

– With the progress of time GP-DO *vis-a-vis* terrestrial organism sector gets aged and become eligible for transfer to the next category. Notwithstanding Category 3 does not have such a provision for further transference for there shall not be complete systemic obliteration without any vestiges when terrestrial organisms (TO) die.

– GP- DO CONCANTENATED WITH THAT OF THE Terrestrial organism(TO) sector when become irretrievable (dead from which no cells can be obtained) are the other outlet that ecelerates the speed of growth of terrestrial organism sector (TO).

– Inflow into category 2 is only from category 1 in the form of transfer of balance of GP RELATIVISTIC to terrestrial organism sector from the category 1.This is evident from the age wise classification scheme. As a result, the speed of growth of category 2 is dependent upon the amount of inflow, which is a function of the quantum of balance of terrestrial organism sector under the category 1.

– The balance of (GP)-terrestrial organism (TO) sector in category 3 is because of transfer of balance from category 2. It is dependent on the amount of terrestrial organism sector under category 2.,THAT CONSUBSTATIATES AND CONCATENATES WITHGREEN PLANTS(see reference).

Notation :

T_{20} : Balance standing in the category 1 of (GP) *vis-a-vis* terrestrial organism

T_{21}: Balance standing in the category 2 of terrestrial organism that corresponds to the concomitant category of green plants.

T_{22}: Balance standing in the category 3 of terrestrial organism with the stratification of green plants

$(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$: Accentuation coefficients

$(b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$: Dissipation coefficients

Formulation of the system :

Under the above assumptions, we derive the following :

The growth speed in category 1 is the sum of two parts:

– A term $(b_{20})^{(3)}T_{21}$ proportional to the amount of balance of GP-terrestrial organisms(TO) in the category 2

– A term $(b'_{20})^{(3)}T_{21}$ representing the quantum of balance dissipated from category 1 .This comprises of GREEN PLANTS -terrestrial organisms which have grown old, qualified to be classified under category 2 and loss of green plants and corresponding terrestrial organisms due to death (dead organic matter- for concatenated equations see end of the paper)

– The growth speed in category 2 is the sum of two parts:

– A term $(b_{21})^{(3)}T_{20}$ constitutive of the amount of inflow from the category 1

– A term $(b'_{21})^{(3)}T_{21}$ the dissipation factor arising due to aging of green plants(GP) coincidental with the terrestrial organism(TO) and the oxygen saved on account of death of green plants and terrestrial organisms. A NOTIONAL chart would spruce up the memory of the whole gamut of concatenation of the Food Cycle.

– The growth speed under category 3 is attributable to inflow from category 2 and oxygen consumption stalled irrevocably and irretrievable due to death of the GP- terrestrial organisms, and hence cannot deplete oxygen quantum in the atmosphere due to cellular respiration any further.

GP-Herbivorous-Carnivorous-DOM (Dead organic matter) - DO (Decomposer organisms) Nutrients - Back to green plants (GP). Notice that respiration takes place due to terrestrial organism (TO) and green plants (GP):

Governing equations:

Following are the differential equations that govern the growth in the terrestrial organisms portfolio

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)} T_{21} - (b'_{20})^{(3)} T_{20} \quad 12$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)} T_{20} - (b'_{21})^{(3)} T_{21} \quad 13$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)} T_{21} - (b'_{22})^{(3)} T_{22} \quad 14$$

$(b_i)^{(3)} > 0$, i = 20, 21, 22 15

$(b_i)^{(3)} > 0$, i = 20, 21, 22 16

$(b_{21})^{(3)} < (b'_{20})^{(3)}$ 17

$(b_{22})^{(3)} < (b'_{21})^{(3)}$ 18

Following the same procedure outlined in the previous section, the general solution of the governing equations is $\alpha'_i Ti + \beta'_i T_i + \gamma'_i T_i = C'_i e_i^{\lambda'_i t}$ i=20, 21, 22, where $C'_{20}, C'_{21}, C'_{22}$ are arbitrary constant coefficients and $\alpha'_{20}, \alpha'_{21}, \alpha'_{22}, \gamma'_{20}, \gamma'_{21}, \gamma'_{22}$ corresponding multipliers to the characteristic roots of the terrestrial organism system.

Nutrients-oxygen consumption (OC) due to cellular respiration dead organic matter (DOM) visa vis green plants (GP) - Terrestrial organism (TO) decomposer organism (DO) - dual system analysis:

In the previous section, we studied the growth of NUTRIENTS (NR) RELATIVISTICALLY with oxygen consumption (OC) due to cellular respiration and GP corresponding to terrestrial organisms separately. In this section, we study the two-portfolio model comprising six-storey nutrients-oxygen consumption due to cellular respiration and green plants -terrestrial organisms.- decomposer organisms. Scheme of age wise classification however remains the same. We make an explicit assumption that only category 2 of green plants-decomposer organisms-terrestrial organisms is responsible for the increase in the dissipation coefficient of the oxygen consumption due to cellular respiration. Terrestrial organisms of three categories dissipating three portfolios of oxygen consumption due to cellular respiration levels follows by mere substitution of corresponding variables. Dissipation coefficients of the terrestrial organism's portfolio are diminished by the contribution of all three categories of nutrients-oxygen consumption due to cellular respiration portfolio of green plants-decomposer organism's terrestrial organisms. This is to facilitate circumvention of the nonlinearity of the equations and consequent unsolvability thereof

We will deonote

– $T_i(t)$, i = 20, 21, 22, the three parts of the GP-DO-terrestrial organisms system analogously to the G_i of the consumption of oxygen due to cellular respiration-DOM-NR takes place due to terrestrial organisms (TO) and green plants (GP).

– By $(a''_i)^{(3)} (T_{21}, t)$ $(T_{21} \geq 0, t \geq 0)$, the contribution of the GP-DO- oxygen consumption (OC) due to cellular respiration of terrestrial organisms takes place due to terrestrial organisms (TO) and green plants (GP) SYSTEM.

- By $(-b''_i)^{(3)} (G_{20}, G_{21}, G_{22}, t) = -(b''_i)^{(3)} (G, t)$, the contribution of the NR-DOM-consumption of oxygen due to cellular respiration to the dissipation coefficient of the NR-DOM-terrestrial organisms SYSTEM.

Governing equations :

The differential system of this model is now

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)} G_{21} - [(a'_{20})^{(3)} + (a''_{20})^{(3)} (T_{21}, t)] G_{20} \quad 19$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)} G_{20} - [(a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}, t)] G_{21} \quad 20$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)} G_{21} - [(a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}, t)] G_{22} \quad 21$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)} T_{21} - [(b'_{20})^{(3)} + (b''_{20})^{(3)} (G_{23}, t)] T_{20} \quad 22$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)} T_{20} - [(b'_{21})^{(3)} + (b''_{21})^{(3)} (G_{23}, t)] T_{21} \quad 23$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)} T_{21} - [(b'_{22})^{(3)} + (b''_{22})^{(3)} (G_{23}, t)] T_{22} \quad 24$$

$+ (a''_{20})^{(3)} (T_{21}, t)$ = First augmentation factor attributable to cellular respiration of terrestrial organism, to the dissipation of oxygen consumption

$- (b''_{20})^{(3)} (G_{23}, t)$ = First detrition factor contributed by oxygen consumption to the dissipation of terrestrial organisms

Where we suppose

– $(a_i)^{(3)}, (a'_i)^{(3)}, (a''_i)^{(3)}, (b_i)^{(3)}, (b'_i)^{(3)}, (b''_i)^{(3)} > 0$, i, j = 20, 21, 22

– The functions $(a''_i)^{(3)}, (b''_i)^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}, (r_i)^{(3)}$:

$$(a''_i)^{(3)} (T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)} \quad 25$$

$$(b''_i)^{(3)} (G, t) \leq (r_i)^{(3)} \leq (b'_i)^{(3)} \leq (\hat{B}_{20})^{(3)} \quad 26$$

$$- \quad \lim_{T_2 \to \infty} (a''_i)^{(3)} (T_{21}, t) = (p_i)^{(3)} \quad 27$$

$$\lim_{G \to \infty} (b''_i)^{(3)} (G, t) = (r_i)^{(3)} \quad 28$$

Definition of : $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$

where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants

and $\boxed{i = 20, 21, 22}$

The satisfy Lipschiz condition :

$$|(a''_i)^{(3)} (T'_{21}, t) - (a''_i)^{(3)} (T_{21}, t)| \leq (\hat{k}_{20})^{(3)} | T_{21} - T'_{21} | e^{-(\hat{M}_{20})^{(3)} t} \quad 29$$

$$|(b''_i)^{(3)} (G, t) - (b''_i)^{(3)} (G, t)| < (\hat{k}_{20})^{(3)} \| G - G' \| e^{-(\hat{M}_{20})^{(3)} t} \quad 30$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a''_i)^{(3)}(T'_{21}, t)$ and $(a''_i)^{(3)}(T_{21}, t)$. (T'_{21}, t) and (T_{21}, t) are points belonging to the interval $[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}]$. It is to be noted that $(a''_i)^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a''_i)^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to terrestrial organisms, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$:

– $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1 \qquad 31$$

– There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}$, $(\hat{k}_{20})^{(3)}$, $(\hat{A}_{20})^{(3)}$ and $(\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a'_i)^{(3)}, (b_i)^{(3)}, (b'_i)^{(3)}, (p_i)^{(3)}, (\gamma)^{(3)}$ i=20, 21, 22 satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a'_i)^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1 \qquad 32$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b'_i)^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1 \qquad 33$$

Theorem 1 :

If the conditions (A) – (E) above are fulfilled, there exists a solution satisfying the conditions

$$Gi(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}, \; Gi(0) = G_i^0 > 0$$

$$Ti(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}, \; Ti(0) = T_i^0 > 0$$

Proof :

Consider operator $\mathbb{A}^{(3)}$ defined on the space of sextuples of continous functions $G_i, T_i : \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0, \; T_i(0) = T_i^0, \; G_i^0 \le (\hat{P}_{20})^{(3)}, \; T_i^0 \le (\hat{Q}_{20})^{(3)} \qquad 34$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t} \qquad 35$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t} \qquad 36$$

By

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t [(a_{20})^{(3)} G_{21}(S_{(20)}) - ((a'_{20})^{(3)} + (a''_{20})^{(3)} (T_{21}(S_{(20)}), S_{(20)})) G_{20}(S_{(20)})] ds_{(20)} \qquad 37$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t [(a_{21})^{(3)} G_{20}(S_{(20)}) - ((a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(S_{(20)}), S_{(20)})) G_{21}(S_{(20)})] ds_{(20)} \qquad 38$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t [(a_{22})^{(3)} G_{21}(S_{(20)}) - ((a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(S_{(20)}), S_{(20)})) G_{22}(S_{(20)})] ds_{(20)} \qquad 39$$

$$\bar{T}_{20}(t) = T_{22}^0 + \int_0^t [(b_{20})^{(3)} T_{21}(S_{(20)}) - (b'_{20})^{(3)} - (b''_{20})^{(3)} (G(S_{(20)}), S_{(20)})) T_{20}(S_{(20)})] ds_{(20)} \qquad 40$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t [(b_{21})^{(3)} T_{20}(S_{(20)}) - (b'_{21})^{(3)} - (b''_{21})^{(3)} (G(S_{(20)}), S_{(20)})) T_{21}(S_{(20)})] ds_{(20)} \qquad 41$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t [(b_{22})^{(3)} T_{21}(S_{(20)}) - (b'_{22})^{(3)} - (b''_{22})^{(3)} (G(S_{(20)}), S_{(20)})) T_{22}(S_{(20)})] ds_{(20)} \qquad 42$$

where $S_{(20)}$ is the integrand over an interval (0, t)

– The operator $\mathbb{A}^{(3)}$ maps the space of functions satisfying 34, 35, 36 into itself. Indeed it is obvious that

$$G_{20}(t) \le G_{20}^0 + \int_0^t (a_{20})^{(3)} (G_{21}^0 + (\hat{P}_{20})(3) e^{(\hat{M}_{20})^{(3)} S_{(20)}})] ds_{(20)} =$$

$$(1 + (a_{20})^{(3)} t) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1 \right) \qquad 43$$

From which it follows that

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0 \right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0} \right)} + (\hat{P}_{20})^{(3)} \right] \qquad 44$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}} \left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0 \right) e^{-\left(\frac{(\hat{P}_{20})^{(3)} + G_j^0}{G_j^0} \right)} \le (\hat{P}_{20})^{(3)} \right] \qquad 45$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{Q}_{20})^{(3)} + T_j^0 \right) e^{-\left(\frac{(\hat{Q}_{20})^{(3)} + T_j^0}{T_j^0} \right)} + (\hat{Q}_{20})^{(3)} \right] \le (\hat{Q}_{20})^{(3)} \qquad 46$$

In order that the operator $\mathbb{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying 34, 35, 36 into itself. The operator $\mathbb{A}^{(3)}$ is a contraction with respect to the metric

$$d\left(((G_{23})^{(1)}, (T_{23})^{(1)}), ((G_{23})^{(2)}, (T_{23})^{(2)}) \right) =$$

$$\sup_t \{ \max_{t \in \mathfrak{R}_+} | G_i^{(1)}(t) - G_i^{(2)}(t) | e^{-(\hat{M}_{20})^{(3)} t}, \max_{t \in \mathfrak{R}_+} | T_t^{(1)}(t) - T_t^{(2)}(t) | e^{-(\hat{M}_{20})^{(3)} t} \} \qquad 47$$

Indeed if we denote

Definition of $\tilde{G}_{23}, \tilde{T}_{23}$:

$$\left((\tilde{G}_{23}),(\tilde{T}_{23})\right)=A^{(3)}\left((G_{23}),(T_{23})\right) \qquad 48$$

It results

$$|\tilde{G}_{20}^{(1)}-\tilde{G}_{20}^{(2)}| \le \int_0^t (a_{20})^{(3)} |G_{21}^{(1)}-G_{21}^{(2)}| e^{-(\hat{M}_{20})^{(3)}S_{(20)}} e^{-(\hat{M}_{20})^{(3)}S_{(20)}ds_{(20)}} + \int_0^t \{(a'_{20})^{(3)} |G_{20}^{(1)}-G_{20}^{(2)}| e^{-(\hat{M}_{20})^{(3)}S_{(20)}} e^{-(\hat{M}_{20})^{(3)}S_{(20)}ds_{(20)}} + (a''_{20})^{(3)}(T_{21}^{(1)},S_{(20)}) |G_{20}^{(1)}-G_{20}^{(2)}| e^{-(\hat{M}_{20})^{(3)}S_{(20)}} e^{-(\hat{M}_{20})^{(3)}S_{(20)}ds_{(20)}} + G_{20}^{(2)} |(a''_{20})^{(3)}(T_{21}^{(1)},S_{(20)}) - (a''_{20})^{(3)}(T_{21}^{(2)},S_{(20)})| e^{-(\hat{M}_{20})^{(3)}S_{(20)}} e^{-(\hat{M}_{20})^{(3)}S_{(20)}ds_{(20)}} \} ds_{(20)} \qquad 49$$

where $S_{(20)}$ represents integrated that is integrated over the interval [0, t]

From the hypotheses on 25, 26, 27, 28 and 29 it follows

$$|G^{(1)}-G^{(2)}| e^{-(\hat{M}_{20})^{(3)}t} \le \frac{1}{(\hat{M}_{20})^{(3)}} \left((a'_{20})^{(3)}+(a'_{20})^{(3)}+(\hat{A}_{20})^{(3)}+(\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}\right) d\left(((G_{23})^{(1)},(T_{23})^{(1)};(G_{23})^{(2)},(T_{23})^{(2)})\right) \qquad 50$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis (34, 35, 36) the result follows

Remark 1: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the soloution bounded by $(\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t}$ and $(\hat{Q}_{20})^{(3)} e^{(\hat{M}(3)_{20})^{(3)}t}$, respectively of R_+.

If instead of proving the existence of the solution on R_+, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(3)}$ and $(b''_i)^{(3)}$, i=20, 21, 22 depend only on T_{21} and respectively on (G_{23}) (and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \ge G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(3)}-(a''_i)^{(3)}(T_{21}(s_{(20)}),S_{(20)})\}ds_{(20)}\right]} \ge 0 \qquad 52$$

$$T_i(t) \ge T_i^0 e^{(-(b'_i)^{(3)}t)} > 0 \text{ for } t > 0$$

Definition of: $((\hat{M}_{20})^{(3)})_1, ((\hat{M}_{20})^{(3)})_2$ and $((\hat{M}_{20})^{(3)})_3$

Remark 3 : If G_{20} is bounded, the same property have also G_{21} and G_{22} indeed if

$G_{20} < (\hat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \le ((\hat{M}_{20})^{(3)})_1 - (a'_{21})^{(3)} G_{21}$

and by integrating 53

$$G_{21} \le ((\hat{M}_{20})^{(3)})_2 = G_{21}^0 + 2(a_{21})^{(3)}((\hat{M}_{20})^{(3)})_1/(a'_{21})^{(3)}$$

In the same way, one can obtain

$$G_{22} \le ((\hat{M}_{20})^{(3)})_3 = G_{22}^0 + 2(a_{22})^{(3)}((\hat{M}_{20})^{(3)})_2/(a'_{22})^{(3)}$$

If G_{21} or G_{22} is bounded, the same property follows for G_{20}, G_{22} and G_{20}, G_{21}, respectively

Remark 4 : If G_{20} is bounded, from below, the same property holds for G_{21} and G_{22}. The proof is analogous with the proceeding one. An analogous property is true if G_{21} is bounded from below : 54

Remarks 5 : If T_{20} is bounded, from below and $\lim_{t\to\infty} ((b''_i)^{(3)}((G_{23})(t), t)) = (b'_{21})^{(3)}$ then $T_{21} \to \infty$ 55

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)} - (b''_i)^{(3)}((G_{23})(t), t) < \varepsilon_3$, $T_{20}(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \ge (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to

$T_{21} \ge \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1-e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$. If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \ge \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)$, $t = \log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded. The same property holds for T_{22} if $\lim_{t\to\infty} ((b''_{22})^{(3)}((G_{23})(t), t)) = (b'_{22})^{(3)}$

We now state a more precise theorem about the bahaviors at infinity of the solutions of equations 37 to 42.

Behaviour of the solutions of equation 37 to 42 56

Theorem 2 : If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

– $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$ 57

$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G, t) + (b''_{21})^{(3)}((G_{23}), t) \le -(\tau_1)^{(3)}$ 58

Definition of $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 59

By $(\nu_1)^{(3)} > 0$, $(\nu_2)^{(3)} < 0$ and, respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations

$(a_{21})^{(3)}(\nu^{(3)})^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$ 60

and $(b_{21})^{(3)}(u^{(3)})^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and 61

By $(\bar{\nu}_1)^{(3)} > 0$, $(\bar{\nu}_2)^{(3)} < 0$ and, respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the 62

roots of the equations $(a_{21})^{(3)}(\nu^{(3)})^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$ 63

and $(b_{21})^{(3)}(u^{(3)})^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ 64

Definition of $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$: 65

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)}$, if $(\nu_0)^{(3)} < (\nu_1)^{(3)}$ 66

$(m_2)^{(3)} = (\bar{\nu}_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}$, if $(\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)}$ 67

and $\boxed{(\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)}$, if $(\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$ 68

and analogously

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, if $(u_0)^{(3)} < (u_1)^{(3)}$ 69

and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$ 70

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, if $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$ 71

Then the solution of 19, 20, 21, 22, 23 and 24 satisfies the inequalities

$$G_{20}^0\, e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20} \le G_{20}^0\, e^{(S_1)^{(3)} t} \quad 72$$

$(p_i)^{(3)}$ is defined by equation 25

$$\frac{1}{(m_1)^{(3)}} G_{20}^0\, e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0\, e^{(S_1)^{(3)} t} \quad 73$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0\, e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{((S_1)^{(3)}} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0\, e^{-(a_{22})^{(3)} t}\right) \quad 74$$

$$\boxed{T_{20}^0\, e(R_1)^{(3)} t \le T_{20}(t) \le T_{20}^0\, e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 75$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0\, e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0\, e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 76$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{((R_1)^{(3)}} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0\, e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} - (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0\, e^{-(R_2)^{(3)} t} \quad 77$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$: 78

where $(S_1)^{(3)} = (a_{20})^{(3)} (m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$ 79

$(R_1)^{(3)} = (b_{20})^{(3)} (\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Proof : From 19, 20, 21, 22, 23, 24 we obtain

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})\nu^{(3)}(T_{21}, t)^{(3)} - (a_{21})^{(3)}\nu^{(3)} \quad 80$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$

It follows

$$-\left((a_{21})^{(3)}(\nu^{(3)})^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \le \frac{d\nu^{(3)}}{dt} \le -\left((a_{21})^{(3)}(\nu^{(3)})^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \quad 81$$

From which one obtains

(a) For $0 > (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \ge \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)}\, e^{[-(a_{21})^{(3)}((\nu_1)^{(3)} - (\nu_0)^{(3)})t]}}{1 + (C)^{(3)}\, e^{[-(a_{21})^{(3)}((\nu_1)^{(3)} - (\nu_0)^{(3)})t]}},$$

$$\boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

It follows $(\nu_0)(3) \le \nu^{(3)}(t) \le (\nu_1)^{(3)}$

In the same manner, we get

$$\nu^{(3)}(t) \ge \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}\, e^{[-(a_{21})^{(3)}((\bar{\nu}_1)^{(3)} - (\bar{\nu}_0)^{(3)})t]}}{1 + (\bar{C})^{(3)}\, e^{[-(a_{21})^{(3)}((\bar{\nu}_1)^{(3)} - (\bar{\nu}_0)^{(3)})t]}},$$

$$\boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}} \quad 82$$

Definition of $(\bar{\nu}_1)^{(3)}$ **:-**

From which we deduce $(\nu_0)^{(3)} \le \nu^{(3)}(t) \le (\bar{\nu}_1)^{(3)}$ 83

(b) If $0 < (\nu_1)^{(3)} (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case,

$$(\nu_1)^{(3)} \ge \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)}\, e^{[-(a_{21})^{(3)}((\nu_1)^{(3)} - (\nu_2)^{(3)})t]}}{1 + (C)^{(3)}\, e^{[-(a_{21})^{(3)}((\nu_1)^{(3)} - (\nu_2)^{(3)})t]}} \le \nu^{(3)}(t) \le \quad 84$$

$$(\nu_1)^{(3)} \ge \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)}\, e^{[-(a_{21})^{(3)}((\nu_1)^{(3)} - (\nu_2)^{(3)})t]}}{1 + (C)^{(3)}\, e^{[-(a_{21})^{(3)}((\nu_1)^{(3)} - (\nu_2)^{(3)})t]}} \le (\bar{\nu}_1)^{(3)}$$

(c) $0 < (\nu_1)^{(3)} \le (\bar{\nu}_1)^{(3)} \le (\bar{\nu}_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$ If we obtain

$$(\nu_1)^{(3)} \le \nu^{(3)}(t) \le \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}\, e^{[-(a_{21})^{(3)}((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)})t]}}{1 + (\bar{C})^{(3)}\, e^{[-(a_{21})^{(3)}((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)})t]}} \le (\nu_0)^{(3)}$$

85

And so with the notation of the first part of condition (c), we have

Definition of $\nu^{(3)}(t)$:

$$(m_2)^{(3)} \leq \nu^{(3)}(t) < (m_1)^{(3)}, \boxed{(\nu)^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}} \qquad 86$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:

$$(\mu_2)^{(3)} \leq u^{(3)}(t) < (\mu_1)^{(3)}, \boxed{(u)^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}} \qquad 87$$

Now, using this result and replacing it in 19, 20, 21, 22, 23 and 24 we get easily the result stated in the theorem.

Paricular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}$, then $(\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20}) = (b''_{21})^{(3)}$, then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$ if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$. This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Stationary solutions and stability:

Stationary solutions and stability curve representative of the variation of oxygen consumption due to cellular respiration of terrestrial organisms *via-a-vis* that of terrestrial organism variation curve lies below the tangent at $G=G_0$ for $G<G_0$ and above the tangent $G>G_0$. Wherever such a situation occurs the point G_0 is called the "point of inflexion". In this case, the tangent has a positive slope that simply means the rate of change of oxygen consumption due to cellular respiration is greater than zero. Above factor shows that it is possible, to draw a curve that has a point of inflexion at a point where the tangent (slope of the curve) is horizontal.

Stationary value :

In all the cases $G=G_0$, $G<G_0$, $G>G_0$ the condition that the rate of change of oxygen value of oxygen consumption is maximum or minimum holds. When this condition holds we have stationary value. We now infer that :

– A necessary and sufficient condition for there to be stationary value of (G) is that the rate of change of oxygen consumption function at G_0 is zero.

– A sufficient conditon for the stationary value at G_0, to be maximum is that the acceleration of the oxygen consumption is less than zero.

– A sufficient condition for the stationary value at G_0, minimum is that acceleration of oxygen consumption is greater than zero.

– With the rate of change of G namely oxygen consumption defined as the accentuation term and the dissipation term, we are sure that the rate of change of oxygen consumption is always positive. 88

– Concept of stationary state is mere methodology although there might be closed system exhibiting symptoms of stationariness.

We can prove the following

Theorem 3 : If $(a''_i)^{(3)}$ and $(b''_i)^{(3)}$ are independent on t, and the conditions (with the notations 25, 26, 27, 28)

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} > 0$

with $(p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation 25 are satisfied, then the system

$(a_{20})^{(3)}G_{21} - [(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})]G_{20} = 0$ 89

$(a_{21})^{(3)}G_{20} - [(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})]G_{21} = 0$ 90

$(a_{22})^{(3)}G_{21} - [(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})]G_{22} = 0$ 91

$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} + (b''_{20})^{(3)}(G_{23})]T_{20} = 0$ 92

$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} + (b''_{21})^{(3)}(G_{23})]T_{21} = 0$ 93

$(b_{20})^{(3)}T_{21} - [(b'_{22})^{(3)} + (b''_{22})^{(3)}(G_{23})]T_{22} = 0$ 94

has a unique positive solution, which is an equilibrium solution for (19 to 24)

Proof :

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)} + (a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$ 95

Definition and uniqueness of T^*_{21} :

After hypothesis $f(0) < 0$, $f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T)$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value, we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T^*_{21})]}, \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T^*_{21})]} \qquad 96$$

– By the same argument, the equations 92, 93 admit solutions G_{20}, G_{21} if

$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - [(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})] + (b''_{20})^{(3)}(G_{23})(b''_{21})(G_{23}) = 0$ 97

where in $G_{23}(G_{20}, G_{21}, G_{21}, G_{22})$ G_{20}, G_{22} must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{21} such that $\varphi(G_{23})^* = 0$

Finally G^*_{21} such that $\varphi((G_{23})^*) = 0$, T^*_{21} given by $f(T^*_{21}) = 0$ and

$$G_{20}^{*}=\frac{(a_{20})^{(3)}G_{21}^{*}}{[(a_{20}^{'})^{(3)}+(a_{20}^{''})^{(3)}(T_{21}^{*})]}\ ,\ G_{22}^{*}=\frac{(a_{22})^{(3)}G_{21}^{*}}{[(a_{20}^{'})^{(3)}+(a_{20}^{''})^{(3)}(T_{21}^{*})]} \quad 98$$

$$T_{20}^{*}=\frac{(b_{20})^{(3)}T_{21}^{*}}{[(b_{20}^{'})^{(3)}-(b_{20}^{''})^{(3)}(G_{23}^{*})]}\ ,\ T_{22}^{*}=\frac{(b_{22})^{(3)}T_{21}^{*}}{[(b_{22}^{'})^{(3)}-(b_{22}^{''})^{(3)}(G_{23}^{*})]} \quad 99$$

Obviously, these values represent an equilibrium solution of 19, 20, 21, 22, 23, 24

Asymptotic stability analysis :

Theorem 4 : If the conditions of the previous theorem are satisfied and if the functions $(a_i^{''})^{(3)}$ and $(b_i^{''})^{(3)}$ belongs to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof : Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:

$$G_i=G_i^{*}+\mathbb{G}_i\ ,\ T_i=T_i^{*}+\mathbb{T}_i \quad 100$$

$$\frac{\partial (a_{21}^{''})^{(3)}}{\partial T_{21}}(T_{21}^{*})=(q_{21})^{(3)}\ ,\ \frac{\partial (b_i^{''})^{(3)}}{\partial G_j}((G_{23})^{*})=S_{ij} \quad 101$$

Then taking into account equations 89 to 94 and neglecting the terms of power 2, we obtain from 19 to 24

$$\frac{d\mathbb{G}_{20}}{dt}=-((a_{20}^{'})^{(3)}+(p_{20})^{(3)})\mathbb{G}_{20}+(a_{20})^{(3)}\mathbb{G}_{21}-(q_{20})^{(3)}G_{20}^{*}\mathbb{T}_{21} \quad 102$$

$$\frac{d\mathbb{G}_{21}}{dt}=-((a_{21}^{'})^{(3)}+(p_{21})^{(3)})\mathbb{G}_{21}+(a_{21})^{(3)}\mathbb{G}_{20}-(q_{21})^{(3)}G_{21}^{*}\mathbb{T}_{21} \quad 103$$

$$\frac{d\mathbb{G}_{22}}{dt}=-((a_{22}^{'})^{(3)}+(p_{22})^{(3)})\mathbb{G}_{22}+(a_{22})^{(3)}\mathbb{G}_{21}-(q_{22})^{(3)}G_{22}^{*}\mathbb{T}_{21} \quad 104$$

$$\frac{d\mathbb{T}_{20}}{dt}=-((b_{20}^{'})^{(3)}-(r_{20})^{(3)})\mathbb{T}_{20}+(b_{20})^{(3)}\mathbb{T}_{21}+(S_{(20)(j)}T_{20}^{*}\mathbb{G}_j) \quad 105$$

$$\frac{d\mathbb{T}_{21}}{dt}=-((b_{21}^{'})^{(3)}-(r_{21})^{(3)})\mathbb{T}_{21}+(b_{21})^{(3)}\mathbb{T}_{20}+(S_{(21)(j)}T_{21}^{*}\mathbb{G}_j) \quad 106$$

$$\frac{d\mathbb{T}_{22}}{dt}=-((b_{22}^{'})^{(3)}-(r_{22})^{(3)})\mathbb{T}_{22}+(b_{22})^{(3)}\mathbb{T}_{21}+(S_{(22)(j)}T_{22}^{*}\mathbb{G}_j) \quad 107$$

The characteristic equation of this system is

$$((\lambda)^{(3)}+(b_{22}^{'})^{(3)}-(r_{22})^{(3)}\{((\lambda(^{(3)}+(a_{22}^{'})^{(3)}+(p_{22})^{(3)})$$
$$[(((\lambda)^{(3)}+(a_{20}^{'})^{(3)}+(p_{20})^{(3)})(q_{21})^{(3)}G_{21}^{*}+(a_{21})^{(3)}(q_{20})^{(3)}G_{20}^{*})]$$
$$(((\lambda)^{(3)}+(b_{20}^{'})^{(3)}-(r_{20})^{(3)}S_{(21),(21)}T_{21}^{*}+(b_{21})^{(3)}S_{(20),(21)}T_{21}^{*})$$
$$+(((\lambda)^{(3)}+(a_{21}^{'})^{(3)}+(p_{21})^{(3)})(q_{20})^{(3)}G_{20}^{*}+(a_{20})^{(3)}(q_{21})^{(1)}G_{21}^{*})$$
$$(((\lambda)^{(3)}(b_{20}^{'})^{(3)}-(r_{20})^{(3)})S_{(21),(20)}T_{21}^{*}+(b_{21})^{(3)}S_{(20),(20)}T_{20}^{*})$$
$$(((\lambda)^{(3)})^2+((a_{20}^{'})^{(3)}+(a_{21}^{'})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}(\lambda)^{(3)})$$
$$(((\lambda)^{(3)})^2+((b_{20}^{'})^{(3)}+(b_{21}^{'})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}(\lambda)^{(3)})$$
$$+(((\lambda)^{(3)})^2+((a_{20}^{'})^{(3)}+(a_{21}^{'})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}(\lambda)^{(3)})(q_{22})^{(3)}G_{22}$$
$$+((\lambda)^{(3)}+(a_{20}^{'})^{(3)}+(p_{20})^{(3)})(a_{22})^{(3)}+(q_{21})^{(3)}G_{21}^{*}+(a_{21})^{(3)}(a_{22})^{(3)}$$
$$(q_{20})^{(3)}G_{20}^{*})$$
$$(((\lambda)^{(3)}+(b_{20}^{'})^{(3)}-(r_{20})^{(3)})S_{(21),(22)}T_{21}^{*}+(b_{21})^{(3)}S_{(20),(22)}T_{21}^{*})\}=0 \quad 108$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

More often than not, models begin with the assumption of 'steady state' and then proceed to trace out the path, which will be followed when the steady state is subjected to some kind of exogenous disturbance. Breathing pattern of terrestrial organisms is another parametric representation to be taken into consideration. It cannot be taken for granted that the sequence generated in this manner will tend to equilibrium *i.e.* a traverse from one steady state to another.

In our model , we have using the tools and techniques by Haimovici, Levin, Volttera, Lotka have brought out implications of steady state, stability, asymptotic stability, behavioral aspects of the solution without any such assumptions, such as those mentioned in the fore going.

In the following, we give equations for the 'dead organic matter-decomposer organism-terrestrial organism-oxygen consumption' system. Solutions and sine-qua-non theoretical aspects are dealt in the next paper (part II)

Governing equations :

Oxygen consumption (OC):

$$\frac{dG_{13}}{dt}=(a_{13})^{(1)}G_{14}-(a_{13}^{'})^{(1)}G_{13} \quad 1a$$

$$\frac{dG_{14}}{dt}=(a_{14})^{(1)}G_{13}-(a_{14}^{'})^{(1)}G_{14} \quad 2a$$

$$\frac{dG_{15}}{dt}=(a_{15})^{(1)}G_{14}-(a_{15}^{'})^{(1)}G_{15} \quad 3a$$

Terrestrial organisms (TO):

$$\frac{dT_{13}}{dt}=(b_{13})^{(1)}T_{14}-(b_{13}^{'})^{(1)}T_{13} \quad 4a$$

$$\frac{dT_{14}}{dt}=(b_{14})^{(1)}T_{13}-(b_{14}^{'})^{(1)}T_{14} \quad 5a$$

$$\frac{dT_{15}}{dt}=(b_{15})^{(1)}T_{14}-(b_{15}^{'})^{(1)}T_{15} \quad 6a$$

Dead organic matter (DOM):

$$\frac{dG_{16}}{dt}=(a_{16})^{(2)}G_{17}-(a_{16}^{'})^{(2)}G_{16} \quad 7a$$

$$\frac{dG_{17}}{dt}=(a_{17})^{(2)}G_{16}-(a_{17}^{'})^{(2)}G_{17} \quad 8a$$

$$\frac{dG_{18}}{dt}=(a_{18})^{(2)}G_{17}-(a_{18}^{'})^{(2)}G_{18} \quad 9a$$

Decomposer organism (DO):

$$\frac{dT_{16}}{dt}=(b_{16})^{(2)}T_{17}-(b_{16}^{'})^{(2)}T_{16} \quad 10a$$

$$\frac{dT_{17}}{dt}=(b_{17})^{(2)}T_{16}-(b_{17}^{'})^{(2)}T_{17} \quad 11a$$

$$\frac{dT_{18}}{dt}=(b_{18})^{(2)}T_{17}-(b_{18}^{'})^{(2)}T_{18} \quad 12a$$

Nutrients :

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)} G_{21} - (a'_{20})^{(3)} G_{20} \quad 13a$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)} G_{20} - (a'_{21})^{(3)} G_{21} \quad 14a$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)} G_{21} - (a'_{22})^{(3)} G_{22} \quad 15a$$

Green plants :

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)} T_{21} - (b'_{20})^{(3)} T_{20} \quad 16a$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)} T_{20} - (b'_{21})^{(3)} T_{21} \quad 17a$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)} T_{21} - (b'_{22})^{(3)} T_{22} \quad 18a$$

Chemical process:

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)} G_{25} - (a'_{24})^{(4)} G_{24} \quad 19a$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)} G_{24} - (a'_{25})^{(4)} G_{25} \quad 20a$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)} G_{25} - (a'_{26})^{(4)} G_{26} \quad 21a$$

Solar radiation:

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)} T_{25} - (b'_{24})^{(4)} T_{24} \quad 22a$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)} T_{24} - (b'_{25})^{(4)} T_{25} \quad 23a$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)} T_{25} - (b'_{26})^{(4)} T_{26} \quad 24a$$

Governing equations of dual concatenated systems terrestrial organisms-oxygen consumption system:

$(-b''_i)^{(1)} (G_{13}, G_{14}, G_{15}, t) = -(b''_i)^{(1)} (G, t)$, i=13, 14, 15 the contribution of the consumption of oxygen due to cellular respiration to the dissipation coefficient of the terrestrial organisms

Oxygen consumption (OC):

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)} G_{14} - \left[(a'_{13})^{(1)} \boxed{+(a''_{13})^{(1)} (T_{14}, t)}\right] G_{13} \quad 25a$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)} G_{13} - \left[(a'_{14})^{(1)} \boxed{+(a''_{14})^{(1)} (T_{14}, t)}\right] G_{14} \quad 26a$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)} G_{14} - \left[(a'_{15})^{(1)} \boxed{+(a''_{15})^{(1)} (T_{14}, t)}\right] G_{15} \quad 27a$$

where $\boxed{+(a''_{13})^{(1)} (T_{14}, t)}$, $\boxed{+(a''_{14})^{(1)} (T_{14}, t)}$, $\boxed{+(a''_{15})^{(1)} (T_{14}, t)}$ are first augmentation coefficients for category 1, 2 and 3 due to terrestrial organism

Terrestrial organisms (TO):

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)} T_{14} - \left[(b'_{13})^{(1)} \boxed{-(b''_{13})^{(1)} (G, t)}\right] T_{13} \quad 28a$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)} T_{13} - \left[(b'_{14})^{(1)} \boxed{-(b''_{14})^{(1)} (G, t)}\right] T_{14} \quad 29a$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)} T_{14} - \left[(b'_{15})^{(1)} \boxed{-(b''_{15})^{(1)} (G, t)}\right] T_{15} \quad 30a$$

where $\boxed{-(b''_{13})^{(1)} (G, t)}$, $\boxed{-(b''_{14})^{(1)} (G, t)}$, $\boxed{-(b''_{15})^{(1)} (G, t)}$ are first detrition coefficients for category 1, 2 and 3 due to oxygen consumption

Dead organic matter-decomposer organism system:

$(-b''_i)^{(2)} (G_{16}, G_{17}, G_{18}, t) = -(b''_i)^{(2)} (G_{19}, t)$, i=16, 17, 18 the contribution of the decomposer for the distegration of dead organic matter

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)} G_{17} - \left[(a'_{16})^{(2)} \boxed{+(a''_{16})^{(2)} (T_{17}, t)}\right] G_{16} \quad 31a$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)} G_{16} - \left[(a'_{17})^{(2)} \boxed{+(a''_{17})^{(2)} (T_{17}, t)}\right] G_{17} \quad 32a$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)} G_{17} - \left[(a'_{18})^{(2)} \boxed{+(a''_{18})^{(2)} (T_{17}, t)}\right] G_{18} \quad 33a$$

where $\boxed{+(a''_{16})^{(2)} (T_{17}, t)}$, $\boxed{+(a''_{17})^{(2)} (T_{17}, t)}$, $\boxed{+(a''_{18})^{(2)} (T_{17}, t)}$ are first augmentation coefficients for category 1, 2 and 3 due to decomposer organism

Decomposer organism (DO):

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)} T_{17} - \left[(b'_{16})^{(2)} \boxed{-(b''_{16})^{(2)} (G_{19}, t)}\right] T_{16} \quad 34a$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)} T_{16} - \left[(b'_{17})^{(2)} \boxed{-(b''_{17})^{(2)} (G_{19}, t)}\right] T_{17} \quad 35a$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)} T_{17} - \left[(b'_{18})^{(2)} \boxed{-(b''_{18})^{(2)} (G_{19}, t)}\right] T_{18} \quad 36a$$

where $\boxed{-(b''_{16})^{(2)} (G_{19}, t)}$, $\boxed{-(b''_{17})^{(2)} (G_{19}, t)}$, $\boxed{-(b''_{18})^{(2)} (G_{19}, t)}$ are first detrition coefficients for category 1, 2 and 3 due to dead organic matter

Green plants *vis a vis* nutrients:

Nutrients:

$(-b''_i)^{(3)} (G_{20}, G_{21}, G_{22}, t) = -(b''_i)^{(3)} (G_{23}, t)$, i=20, 21, 22

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)} G_{21} - \left[(a'_{20})^{(3)} \boxed{+(a''_{20})^{(3)} (T_{21}, t)}\right] G_{20} \quad 37a$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)} G_{20} - \left[(a'_{21})^{(3)} \boxed{+(a''_{21})^{(3)} (T_{21}, t)}\right] G_{21} \quad 38a$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)} G_{21} - \left[(a'_{22})^{(3)} \boxed{+(a''_{22})^{(3)} (T_{22}, t)}\right] G_{22} \quad 39a$$

where $\boxed{+(a''_{20})^{(3)} (T_{21}, t)}$, $\boxed{+(a''_{21})^{(3)} (T_{21}, t)}$, $\boxed{+(a''_{22})^{(3)} (T_{21}, t)}$ are first augmentation coefficients for category 1, 2 and 3 due to plants

Green plants :

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)} T_{21} - \left[(b'_{20})^{(3)} \boxed{-(b''_{20})^{(3)} (G_{23}, t)}\right] T_{20} \quad 40a$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)} T_{20} - \left[(b'_{21})^{(3)} \boxed{-(b''_{21})^{(3)}(G_{23},t)} \right] T_{21} \quad 41a$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)} T_{21} - \left[(b'_{22})^{(3)} \boxed{-(b''_{22})^{(3)}(G_{23},t)} \right] T_{22} \quad 42a$$

where $\boxed{-(b''_{20})^{(1)}(G_{20},t)}$, $\boxed{-(b''_{21})^{(1)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(1)}(G_{23},t)}$

are first detrition coefficients for category 1, 2 and 3 due to nutrients

Chemical process v/s solar radiation-solar radiation dissipates chemical chemical process (lack of photosynthesis) (inside sun also chemical process may be affected due to sun cycles)

Chemical process :

$(-b''_i)^{(4)}(G_{24}, G_{25}, G_{26}, t) = -(b''_i)^{(3)}(G_{27}, t)$, i=24, 25, 26

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)} G_{25} - \left[(a'_{24})^{(4)} \boxed{+(a''_{24})^{(4)}(T_{25},t)} \right] G_{24} \quad 43a$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)} G_{24} - \left[(a'_{25})^{(4)} \boxed{+(a''_{25})^{(4)}(T_{25},t)} \right] G_{25} \quad 44a$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)} G_{25} - \left[(a'_{26})^{(4)} \boxed{+(a''_{26})^{(4)}(T_{26},t)} \right] G_{26} \quad 45a$$

Solar radiation:

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)} T_{25} - \left[(b'_{24})^{(4)} \boxed{-(b''_{24})^{(4)}(G_{27},t)} \right] T_{24} \quad 46a$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)} T_{24} - \left[(b'_{25})^{(4)} \boxed{-(b''_{25})^{(4)}(G_{27},t)} \right] T_{25} \quad 47a$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)} T_{25} - \left[(b'_{26})^{(4)} \boxed{-(b''_{26})^{(4)}(G_{27},t)} \right] T_{26} \quad 48a$$

$+(a''_{24})^{(4)}(T_{25}, t)$ = First augmentation factor attributable to solar radiation

$-(b''_{24})^{(4)}(G, t)$ = First detrition factor contributed by chemical process

Governing equations of concatenated system of two concatenated dual system:

Terrestrial organisms-Dead organic matter system

Dead organic matter dissipates terrestrial organism-Contagion/ Pestilence

Dead organic matter (DOM):

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)} G_{17} - \left[(a'_{16})^{(2)} \boxed{+(a''_{16})^{(2)}(T_{17},t)} \boxed{-(a''_{13})^{(1,1)}(T_{14},t)} \right] G_{16} \quad 49a$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)} G_{16} - \left[(a'_{17})^{(2)} \boxed{+(a''_{17})^{(2)}(T_{17},t)} \boxed{-(a''_{14})^{(1,1)}(T_{14},t)} \right] G_{17} \quad 50a$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)} G_{17} - \left[(a'_{18})^{(2)} \boxed{+(a''_{18})^{(2)}(T_{17},t)} \boxed{-(a''_{15})^{(1,1)}(T_{14},t)} \right] G_{18} \quad 51a$$

where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$

are first augmentation coefficients for category 1, 2 and 3 due to decomposer organism

$\boxed{-(a''_{13})^{(1,1)}(T_{14},t)}$, $\boxed{-(a''_{14})^{(1,1)}(T_{14},t)}$, $\boxed{-(a''_{15})^{(1,1)}(T_{14},t)}$ are second detrition coefficients for category 1, 2 and 3 due to terrestrial organisms

Terrestrial organisms (TO):

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)} T_{14} - \left[(b'_{13})^{(1)} \boxed{-(b''_{13})^{(1)}(G,t)} \boxed{+(b''_{16})^{(2,2)}(G_{19},t)} \right] T_{13} \quad 52a$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)} T_{13} - \left[(b'_{14})^{(1)} \boxed{-(b''_{14})^{(1)}(G,t)} \boxed{+(b''_{17})^{(2,2)}(G_{19},t)} \right] T_{14} \quad 53a$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)} T_{14} - \left[(b'_{15})^{(1)} \boxed{-(b''_{15})^{(1)}(G,t)} \boxed{+(b''_{18})^{(2,2)}(G_{19},t)} \right] T_{15} \quad 54a$$

where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3 due to oxygen consumption and

$\boxed{+(b''_{16})^{(2,2)}(G_{19},t)}$, $\boxed{+(b''_{17})^{(2,2)}(G_{19},t)}$, $\boxed{+(b''_{18})^{(2,2)}(G_{19},t)}$

are second augmentation coefficients for category 1, 2 and 3 due to dead organic matter

Oxygen consumption (OC):

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)} G_{14} - \left[(a'_{13})^{(1)} \boxed{+(a''_{13})^{(1)}(T_{14},t)} \right] G_{13} \quad 55a$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)} G_{13} - \left[(a'_{14})^{(1)} \boxed{+(a''_{14})^{(1)}(T_{14},t)} \right] G_{14} \quad 56a$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)} G_{14} - \left[(a'_{15})^{(1)} \boxed{+(a''_{15})^{(1)}(T_{14},t)} \right] G_{15} \quad 57a$$

where $\boxed{+(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1)}(T_{14},t)}$

are first augmentation coefficients for category 1, 2 and 3 due to terrestrial organism

Decomposer organism (DO):

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)} T_{17} - \left[(b'_{16})^{(2)} \boxed{-(b''_{16})^{(2)}(G_{19},t)} \right] T_{16} \quad 58a$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)} T_{16} - \left[(b'_{17})^{(2)} \boxed{-(b''_{17})^{(2)}(G_{19},t)} \right] T_{17} \quad 59a$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)} T_{17} - \left[(b'_{18})^{(2)} \boxed{-(b''_{18})^{(2)}(G_{19},t)} \right] T_{18} \quad 60a$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$

are first detrition coefficients for category 1, 2 and 3 due to dead organic matter

Decomposer organism dissipates chemical process:

Chemical process:

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)} G_{25} - \left[(a'_{24})^{(4)} \boxed{+(a''_{24})^{(4)}(T_{25},t)} \boxed{+(a''_{16})^{(2,2)}(T_{17},t)} \right] G_{24} \quad 61a$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)} G_{24} - \left[(a'_{25})^{(4)} \boxed{+(a''_{25})^{(4)}(T_{25},t)} \boxed{+(a''_{17})^{(2,2)}(T_{17},t)} \right] G_{25} \tag{62a}$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)} G_{25} - \left[(a'_{26})^{(4)} \boxed{+(a''_{26})^{(4)}(T_{25},t)} \boxed{+(a''_{18})^{(2,2)}(T_{17},t)} \right] G_{26} \tag{63a}$$

$\boxed{+(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4)}(T_{25},t)}$ are first augmentation coefficients for category 1, 2 and 3

, , are second augmentation coefficients for category 1, 2 and 3 due to decomposer organism

Decomposer organism:

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)} T_{17} - \left[(b'_{16})^{(2)} \boxed{-(b''_{16})^{(2)}(G_{19},t)} \boxed{-(b''_{24})^{(4,4)}(G_{27},t)} \right] T_{16} \tag{64a}$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)} T_{16} - \left[(b'_{17})^{(2)} \boxed{-(b''_{17})^{(2)}(G_{19},t)} \boxed{-(b''_{25})^{(4,4)}(G_{27},t)} \right] T_{17} \tag{65a}$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)} T_{17} - \left[(b'_{18})^{(2)} \boxed{-(b''_{18})^{(2)}(G_{19},t)} \boxed{-(b''_{26})^{(4,4)}(G_{23},t)} \right] T_{18} \tag{66a}$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3 due to dead organic matter

$\boxed{-(b''_{20})^{(3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3)}(G_{23},t)}$ are second detrition coefficients for category 1, 2 and 3 due to chemical process

Terrestrial organism dissipates nutrients:

Nutrients:

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)} G_{21} - \left[(a'_{20})^{(3)} \boxed{+(a''_{20})^{(3)}(T_{21},t)} \boxed{+(a''_{13})^{(1,1)}(T_{14},t)} \right] G_{20} \tag{67a}$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)} G_{20} - \left[(a'_{21})^{(3)} \boxed{+(a''_{21})^{(3)}(T_{21},t)} \boxed{+(a''_{13})^{(1,1)}(T_{14},t)} \right] G_{21} \tag{68a}$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)} G_{21} - \left[(a'_{22})^{(3)} \boxed{+(a''_{22})^{(3)}(T_{21},t)} \boxed{+(a''_{15})^{(1,1)}(T_{14},t)} \right] G_{22} \tag{69a}$$

where $\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3 to plants

$\boxed{-(a''_{13})^{(1,1)}(T_{14},t)}$, $\boxed{-(a''_{14})^{(1,1)}(T_{14},t)}$, $\boxed{-(a''_{15})^{(1,1)}(T_{14},t)}$

are second detrition coefficients for category 1, 2 and 3 due to terrestrial organism

Terrestrial organism:

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)} T_{14} - \left[(b'_{13})^{(1)} \boxed{-(b''_{13})^{(1)}(G,t)} \boxed{-(b''_{20})^{(3,3)}(G_{23},t)} \right] T_{13} \tag{70a}$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)} T_{13} - \left[(b'_{14})^{(1)} \boxed{-(b''_{14})^{(1)}(G,t)} \boxed{-(b''_{21})^{(3,3)}(G_{23},t)} \right] T_{14} \tag{71a}$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)} T_{14} - \left[(b'_{15})^{(1)} \boxed{-(b''_{15})^{(1)}(G,t)} \boxed{-(b''_{21})^{(3,3)}(G_{23},t)} \right] T_{15} \tag{72a}$$

where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first augmentation coefficients for category 1, 2 and 3 to oxygen consumption.

where $\boxed{-(b''_{20})^{(3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3)}(G_{23},t)}$ are second detrition coefficients for category 1, 2 and 3 due to nutrients.

Plants dissipate dead organic matter:

Dead organic matter:

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)} G_{17} - \left[(a'_{16})^{(2)} \boxed{+(a''_{16})^{(2)}(T_{17},t)} \boxed{+(a''_{20})^{(3,3)}(T_{21},t)} \right] G_{16} \tag{73a}$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)} G_{16} - \left[(a'_{17})^{(2)} \boxed{+(a''_{17})^{(2)}(T_{17},t)} \boxed{+(a''_{21})^{(3,3)}(T_{21},t)} \right] G_{17} \tag{74a}$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)} G_{17} - \left[(a'_{18})^{(2)} \boxed{+(a''_{18})^{(2)}(T_{17},t)} \boxed{+(a''_{22})^{(3,3)}(T_{22},t)} \right] G_{18} \tag{75a}$$

where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3 due to decomposer organism

where $\boxed{+(a''_{20})^{(3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3)}(T_{17},t)}$, $\boxed{+(a''_{22})^{(3,3)}(T_{21},t)}$ are second augmentation coefficients for category 1, 2 and 3 due to plants

Plants:

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)} T_{21} - \left[(b'_{20})^{(3)} \boxed{-(b''_{20})^{(3)}(G_{23},t)} \boxed{-(b''_{16})^{(2,2)}(G_{19},t)} \right] T_{20} \tag{76a}$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)} T_{20} - \left[(b'_{21})^{(3)} \boxed{-(b''_{21})^{(3)}(G_{23},t)} \boxed{-(b''_{17})^{(2,2)}(G_{19},t)} \right] T_{21} \tag{77a}$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)} T_{21} - \left[(b'_{22})^{(3)} \boxed{-(b''_{22})^{(3)}(G_{23},t)} \boxed{-(b''_{18})^{(2,2)}(G_{19},t)} \right] T_{22} \tag{78a}$$

where $\boxed{-(b''_{20})^{(1)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(1)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(1)}(G_{23},t)}$

are first augmentation coefficients for category 1, 2 and 3 due to nutrients

$\boxed{-(b''_{16})^{(2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2)}(G_{19},t)}$

are second detrition coefficients for category 1, 2 and 3 due to dead organic matter

Decomposer organism dissipates nutrients:

Nutrients:

$$\frac{dG_{20}}{dt}=(a_{20})^{(3)}G_{21}-\left[(a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)}\boxed{+(a''_{16})^{(2,2)}(T_{17},t)}\right]G_{20}\qquad 79a$$

$$\frac{dG_{21}}{dt}=(a_{21})^{(3)}G_{20}-\left[(a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)}\boxed{+(a''_{17})^{(2,2)}(T_{17},t)}\right]G_{21}\qquad 80a$$

$$\frac{dG_{22}}{dt}=(a_{22})^{(3)}G_{21}-\left[(a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)}\boxed{+(a''_{18})^{(2,2)}(T_{17},t)}\right]G_{22}\qquad 81a$$

where $\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$

are first augmentation coefficients for category 1, 2 and 3 to plants

$\boxed{-(a''_{16})^{(2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2)}(T_{17},t)}$, $\boxed{-(a''_{17})^{(2,2)}(T_{17},t)}$

are second detrition coefficients for category 1, 2 and 3 due to decomposer organism

Decomposer organism:

$$\frac{dT_{16}}{dt}=(b_{16})^{(2)}T_{17}-\left[(b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)}-\boxed{(b''_{20})^{(3,3)}(G_{23},t)}\right]T_{16}\qquad 82a$$

$$\frac{dT_{17}}{dt}=(b_{17})^{(2)}T_{16}-\left[(b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)}-\boxed{(b''_{21})^{(3,3)}(G_{23},t)}\right]T_{17}\qquad 83a$$

$$\frac{dT_{18}}{dt}=(b_{18})^{(2)}T_{17}-\left[(b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)}-\boxed{(b''_{22})^{(3,3)}(G_{23},t)}\right]T_{18}\qquad 84a$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$

are first detrition coefficients for category 1, 2 and 3 due to dead organic matter

$\boxed{-(b''_{20})^{(3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3)}(G_{23},t)}$

are second detrition coefficients for category 1, 2 and 3 due to nutrients

Oxygen consumption-Decomposer organism system decomposer organism dissipates oxygen consumption :

Decomposer organism (DO):

$$\frac{dT_{16}}{dt}=(b_{16})^{(2)}T_{17}-\left[(b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)}\boxed{-(b''_{13})^{(1,1)}(G,t)}\right]T_{16}\qquad 85a$$

$$\frac{dT_{17}}{dt}=(b_{17})^{(2)}T_{16}-\left[(b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)}\boxed{-(b''_{14})^{(1,1)}(G,t)}\right]T_{17}\qquad 86a$$

$$\frac{dT_{6}}{dt}=(b_{18})^{(2)}T_{17}-\left[(b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)}\boxed{-(b''_{15})^{(1,1)}(G,t)}\right]T_{18}\qquad 87a$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$

are first detrition coefficients for category 1, 2 and 3 due to dead organic matter

$\boxed{-(b''_{13})^{(1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1)}(G,t)}$ are

second detrition coefficients for category 1, 2 and 3 due to oxygen consumption

Oxygen conusmption (OC):

$$\frac{dG_{13}}{dt}=(a_{13})^{(1)}G_{14}-\left[(a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2)}(T_{17},t)}\right]G_{13}\qquad 88a$$

$$\frac{dG_{14}}{dt}=(a_{14})^{(1)}G_{13}-\left[(a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2)}(T_{17},t)}\right]G_{14}\qquad 89a$$

$$\frac{dG_{15}}{dt}=(a_{15})^{(1)}G_{14}-\left[(a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2)}(T_{17},t)}\right]G_{15}\qquad 90a$$

where $\boxed{+(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1)}(T_{14},t)}$

are first augmentation coefficients for category 1, 2 and 3 to terrestrial organism

$\boxed{+(a''_{16})^{(2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2)}(T_{17},t)}$

are second detrition coefficients for category 1, 2 and 3 due to decomposer organism

Dead organic matter (DOM):

$$\frac{dG_{16}}{dt}=(a_{16})^{(2)}G_{17}-\left[(a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)}\right]G_{16}\qquad 91a$$

$$\frac{dG_{17}}{dt}=(a_{17})^{(2)}G_{16}-\left[(a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)}\right]G_{17}\qquad 92a$$

$$\frac{dG_{18}}{dt}=(a_{18})^{(2)}G_{17}-\left[(a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)}\right]G_{18}\qquad 93a$$

where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$

are first augmentation coefficients for category 1, 2 and 3 due to decomposer organism

Terrestrial organisms (TO):

$$\frac{dT_{13}}{dt}=(b_{13})^{(1)}T_{14}-\left[(b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)}\right]T_{13}\qquad 94a$$

$$\frac{dT_{14}}{dt}=(b_{14})^{(1)}T_{13}-\left[(b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)}\right]T_{14}\qquad 95a$$

$$\frac{dT_{15}}{dt}=(b_{15})^{(1)}T_{14}-\left[(b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)}\right]T_{15}\qquad 96a$$

where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3 due to oxygen consumption

Plants dissipate oxygen consumption:

Oxygen conusmption (OC):

$$\frac{dG_{13}}{dt}=(a_{13})^{(1)}G_{14}-\left[(a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\boxed{+(a''_{16})^{(3,3)}(T_{21},t)}\right]G_{13}$$ 97a

$$\frac{dG_{14}}{dt}=(a_{14})^{(1)}G_{13}-\left[(a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\boxed{+(a''_{21})^{(3,3)}(T_{17},t)}\right]G_{14}$$ 98a

$$\frac{dG_{15}}{dt}=(a_{15})^{(1)}G_{14}-\left[(a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\boxed{+(a''_{22})^{(3,3)}(T_{21},t)}\right]G_{15}$$ 99a

where $\boxed{+(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3 to terrestrial organism

$\boxed{+(a''_{20})^{(3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3)}(T_{21},t)}$ are second detrition coefficients for category 1, 2 and 3 due to decomposer organism

Plants:

$$\frac{dT_{20}}{dt}=(b_{20})^{(3)}T_{21}-\left[(b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)}\boxed{-(b''_{13})^{(1,1)}(G,t)}\right]T_{20}$$ 100a

$$\frac{dT_{21}}{dt}=(b_{21})^{(3)}T_{20}-\left[(b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)}\boxed{-(b''_{14})^{(1,1)}(G,t)}\right]T_{21}$$ 101a

$$\frac{dT_{22}}{dt}=(b_{22})^{(3)}T_{21}-\left[(b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)}\boxed{-(b''_{15})^{(1,1)}(G,t)}\right]T_{22}$$ 102a

where $\boxed{-(b''_{20})^{(1)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(1)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(1)}(G_{23},t)}$ are first augmentation coefficients for category 1, 2 and 3 due to nutrients

$\boxed{-(b''_{13})^{(1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1)}(G,t)}$ are second detrition coefficients for category 1, 2 and 3 due to oxygen consumption

Nutrients :

$$\frac{dG_{20}}{dt}=(a_{20})^{(3)}G_{21}-\left[(a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)}\right]G_{20}$$ 103a

$$\frac{dG_{21}}{dt}=(a_{21})^{(3)}G_{20}-\left[(a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)}\right]G_{21}$$ 104a

$$\frac{dG_{22}}{dt}=(a_{22})^{(3)}G_{21}-\left[(a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{22},t)}\right]G_{22}$$ 105a

where $\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3 due to plants

Decomposer organism dissipates oxygen consumption:

Terrestrial organisms dissipates dead organic matter:

Dead organic matter (DOM):

$$\frac{dG_{16}}{dt}=(a_{16})^{(2)}G_{17}-\left[(a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)}\boxed{+(a''_{13})^{(1,1,1)}(T_{14},t)}\right]G_{16}$$ 106a

$$\frac{dG_{17}}{dt}=(a_{17})^{(2)}G_{16}-\left[(a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)}\boxed{+(a''_{14})^{(1,1,1)}(T_{14},t)}\right]G_{17}$$ 107a

$$\frac{dG_{18}}{dt}=(a_{18})^{(2)}G_{17}-\left[(a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)}\boxed{+(a''_{15})^{(1,1,1)}(T_{14},t)}\right]G_{18}$$ 108a

where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3 due to decomposer organism

and $\boxed{+(a''_{13})^{(1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1)}(T_{14},t)}$ are second augmentation coefficients for category 1, 2 and 3 due to terrestrial organisms

Terrestrial organisms (TO):

$$\frac{dT_{13}}{dt}=(b_{13})^{(1)}T_{14}-\left[(b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)}\boxed{+(b''_{16})^{(2,2,2)}(G_{19},t)}\right]T_{13}$$ 109a

$$\frac{dT_{14}}{dt}=(b_{14})^{(1)}T_{13}-\left[(b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)}\boxed{+(b''_{17})^{(2,2,2)}(G_{19},t)}\right]T_{14}$$ 110a

$$\frac{dT_{15}}{dt}=(b_{15})^{(1)}T_{14}-\left[(b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)}\boxed{+(b''_{18})^{(2,2,2)}(G_{19},t)}\right]T_{15}$$ 111a

where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first augmentation coefficients for category 1, 2 and 3 due to oxygen consumption

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3 due to dead organic matter

Oxygen conusmption (OC):

$$\frac{dG_{13}}{dt}=(a_{13})^{(1)}G_{14}-\left[(a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}\right]G_{13}$$ 112a

$$\frac{dG_{14}}{dt}=(a_{14})^{(1)}G_{13}-\left[(a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}\right]G_{14}$$ 113a

$$\frac{dG_{15}}{dt}=(a_{15})^{(1)}G_{14}-\left[(a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\boxed{+(a''_{22})^{(2,2,2)}(T_{21},t)}\right]G_{15}$$

114a

where $\boxed{+(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3 to terrestrial organism

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$ are second detrition coefficients for category 1, 2 and 3 due to decomposer organism

Decomposer organism (DO):

$$\frac{dT_{16}}{dt}=(b_{16})^{(2)}T_{17}-\left[(b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)}\boxed{-(b''_{13})^{(1,1,1)}(G,t)}\right]T_{16}$$

115a

$$\frac{dT_{17}}{dt}=(b_{17})^{(2)}T_{16}-\left[(b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)}\boxed{-(b''_{14})^{(1,1,1)}(G,t)}\right]T_{17}$$

116a

$$\frac{dT_{18}}{dt}=(b_{18})^{(2)}T_{17}-\left[(b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)}\boxed{-(b''_{15})^{(1,1,1)}(G,t)}\right]T_{18}$$

117a

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3 due to dead organic matter

$\boxed{-(b''_{13})^{(1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1)}(G,t)}$ are second detrition coefficients for category 1, 2 and 3 due to oxygen consumption

REFERENCES

Haimovici, A. (1982). On the growth of a two species ecological system divided on age groups. Tensor, Vol. 37 (1982), Commemoration volume dedicated to Professor Akitsugu Kawaguchi on his 80th birthday

Capra, Frtjof "The web of life" Flamingo, Harper Collins See "Dissipative structures" pages 172-188

Heylighen, F. (2001). The science of self-organization and adaptivity, In : L.D. Kiel, (ed). Knowledge Management, Organizational Intelligence and Learning, and Complexity, In: *The encyclopedia of life support systems* ((EOLSS), (Eolss Publishers, Oxford) [http://www.eolss.net

Matsui, T, Masunaga, H., Kreidenweis, S.M., Pielke Sr., R.A., Tao, W.K., Chin, M. and Kaufman, Y.J. (2006). Satellite-based assessment of marine low cloud variability associated with aerosol, atmospheric stability, and the diurnal cycle, *J. Geophys. Res.*, 111, D17204,doi:10.1029/2005JD006097

Stevens, B., Feingold, G., Cotton, W.R. and Walko, R.L. Elements of the microphysical structure of numerically simulated nonprecipitating stratocumulus. *J. Atmos. Sci.*, **53** : 980-1006

Feingold, G., Koren, I., Wang, H.L., Xue, H.W. and Brewer, W.A. (2010). Precipitation-generated oscillations in open cellular cloud fields" *Nature*, *466* (7308) 849-852, doi: 10.1038/nature093[21], Published 12-Aug 2010

Wood, R. (2006). The rate of loss of cloud droplets by coalescence in warm clouds" J.Geophys. Res., 111, doi: 10.1029/2006JD007553, 2006

Rund, H. (1959). The differential geometry of finsler spaces, Grund. Math. Wiss. Springer-Verlag, Berlin, 1959

Dold, A. (1972). Lectures on algebraic topology, 1972, Springer-Verlag

Levin, S. (1976). Some Mathematical questions in Biology vii, Lectures on Mathematics in life sciences, vol 8" The American Mathematical society, Providence, Rhode island 1976.

Received : *19.04.2012;* **Revised :** *08.05.2012;* **Accepted :** *23.05.2012*