IJCBM International Journal of Commerce and Business Management
e ISSN–0976–7940 Volume **8** | Issue 1 | April, 2015 | 1-6
DOI: 10.15740/HAS/IJCBM/8.1/1-6
Visit us : ***www.researchjournal.co.in***

RESEARCH PAPER

Selecting the best alternatives of multi - criteria decision making problem based on fuzzy TOPSIS method

VIKAS S. JADHAV, L.V. SUKANE AND V.H. BAJAJ

Received : *24.06.2014;* **Revised :** *01.02.2015;* **Accepted :** *15.02.2015*

ABSTRACT

In this paper, we propose Multi Criteria Decision Making (MCDM) problem is one of the famous different kind of decision making problem. In more cases in real situations, determining the exact values for MCDM problems is difficult or impossible. So, the value of alternatives with respect to the criteria or / and the values of weights, are considered as fuzzy values (fuzzy numbers). In such conditions, the conventional crisp approaches for solving MCDM problems tend to be less effective for dealing with the imprecise or vagueness nature of the linguistic assessment. In this situation, the fuzzy MCDM method are applied for solving MCDM problems. Here, fuzzy TOPSIS (Technique for order preference by similarity to ideal solution) method based on a fuzzy distance measure is used in which the distance from the Fuzzy Positive Ideal Solution (FPIS) and Fuzzy Negative Ideal Solution (FNIS) are calculated. The resulted distances were used to calculate the similarity to ideal solution. Later an optimal membership degree (Closeness co-efficient) of each alternative is computed to estimate to which extent an alternative belongs to both FPIS and FNIS. The closer the degree of membership of FPIS and the farther from FNIS the more preferred the alternative. The membership degree is obtained by the optimization of a defined objective function that measures the degree of which an alternative is similar / dissimilar to the ideal solutions. A numerical example is also demonstrate the procedure of the proposed fuzzy TOPSIS method in the decision making processes.

KEY WORDS : Single machine, Processing time, Customer due dates, Total penalty cost, Genetic algorithm (GA)

How to cite this paper : Jadhav, Vikas S., Sukane, L.V. and Bajaj, V.H. (2015). Selecting the best alternatives of multi - criteria decision making problem based on fuzzy TOPSIS method. *Internat. J. Com. & Bus. Manage,* **8**(1) : 1-6.

TOPSIS (Technique for order preference by similarity to ideal solution) method is a popular approach to Multi-Criteria Decision Making problem that was proposed by (Chang, 1992). According to this technique, the best alternative would be the one that is nearest to the Positive Ideal Solution (PIS) and the farthest from the Negative Ideal Solution (NIS) for solving a MCDM problem. In short, the positive ideal solution is composed of all best values attainable of criteria; whereas the negative ideal solution is made up of all worst values attainable of criteria. In this method two artificial alternatives are hypothesized (Chen, 2002 and Jadhav and Bajaj, 2013).

Positive ideal solution : The one which has the best level for all attributes considered.

Negative ideal solution : The one which has the worst attribute values.

TOPSIS selects the alternative that is the closest to the ideal solution and farthest from negative ideal alternative.

This method has been widely used in the literature. Some papers in the literature applied the TOPSIS method for solving

MEMBERS OF THE RESEARCH FORUM

Correspondence to:
VIKAS S. JADHAV, Department of Statistics, A.R.A.C. Science College, VAIBHAVWADI (M.S.) INDIA
Email: vsjadhavstats@gmail.com

Authors' affiliations:
L.V. SUKANE, Department of Statistics, Krishna Mahavidhalay, Rethare (B.K.), KARAD (M.S.) INDIA
Email: lvsukane@gmail.com

V.H. BAJAJ, Department of Statistics, Dr. Babasaheb Ambedkar Marathwada University, AURANGABAD (M.S.) INDIA
Email: vhbajaj@gmail.com

real application problems. For example: (Chen, 2002 and Chu, 2002). Multi-Criteria Decision Making (MCDM) is the process to define the ranking of all possible alternatives respective to the goal and criteria. In real-life applications of MCDM method, data are usually imprecise, uncertain and/or vague. In such applications, decision makers usually give preferences in linguistic variables and linguistic variables will be then converted to Fuzzy number for further evaluation. The fuzzy set theory is an efficient way to model uncertainty and imprecision in terms of linguistic variable. From concepts of MCDM method of Analytical Hierarchy Process (AHP), (Hwang and Yoon, 1981), and others have developed the Fuzzy AHP to handle the fuzziness in decision making (Lai *et al.*, 1994 and Liu and Liu, 2010).

The aim of this paper is to propose a new MCDM method (Fuzzy-TOPSIS) to deal with linguistic preferences in a Fuzzy environment. The decision making problem is presented in hierarchical structure similar to those in the AHP method. Calculating priority vector of criteria which is presented as an optimization problem can be solved by using FTOPSIS to find the priority vector, which maximizes the Triangular Membership Function (TMFs). The ranking of alternatives is then defined by the TOPSIS method in terms of calculating the Fuzzy distance among ideal alternative and other alternatives. Proposed TOPSIS method utilizes the advantages of Fuzzy set theory, and TOPSIS, allows the decision making processes to become realistic and effective.

Linguistic variable and fuzzy number :

Linguistic variable :

A linguistic variable is a "variable whose values are not numbers but words or sentences in a natural or artificial language" (Mikhailov and Tsvetinov, 2004). Using linguistic values (words or sentences) expresses less specific than numerical ones, but it is closely related to the way that humans express and use their knowledge. In order to deal with the uncertainty and vagueness in the linguistic evaluation, many researchers have applied Fuzzy Set Theory to convert linguistic variable to fuzzy number (Jadhav and Bajaj, 2010) proposed "Triangular Fuzzy Expression of Linguistic Variable" as follows:

Suppose S is a set of ordered natural linguistic label which is consisted of odd elements k. Let

$$S = s_0, s_1, \ldots, s_{k-1}$$

and the Triangular Fuzzy Expression of Linguistic Variable is :

$$S_i = (S_i^l, s_i^m, s_i^u),$$

then :

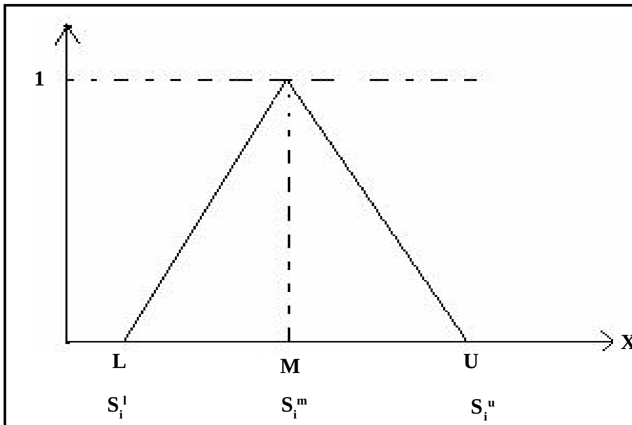

Fig. 1: Triangular Fuzzy Number $S_i = (S_i^l, s_i^m, s_i^u)$

s_i^l, s_i^m and s_i^u are defined as follows :

$$\begin{cases} s_i^0 = 0 \\ s_i^l = \dfrac{i-1}{k-1} \quad (1 \le i \le k-1) \\ s_i^m = \dfrac{i}{k-1} \quad (0 \le i \le k-1) \\ s_i^u = \dfrac{i+1}{k-1} \quad (0 \le i \le k-2) \\ s_{k-1}^u = 1 \end{cases} \qquad \text{..........(1)}$$

By applying equation (1), linguistic variable is converted to triangular fuzzy numbers (TFNs) for corresponding fuzzy label. Table 1 shows converting seven-linguistic expression to triangular fuzzy numbers while Fig. 2 shows seven-linguistic variables with triangular fuzzy membership function. Meanwhile Table 2 shows the conversion of nine-linguistic expressions to triangular fuzzy numbers (TFNs) and Fig. 3

Table 1 : Converting seven-linguistic expressions to triangular fuzzy numbers

Fuzzy label	Fuzzy linguistic expression	TFNs
S_0	Very poor	(0, 0, 0.167)
S_1	Poor	(0, 0.167, 0.333)
S_2	Moderately poor	(0.67, 0.333, 0.5)
S_3	Fair	(0.333, 0.5, 0.667)
S_4	Moderately good	(0.5, 0.667, 0.833)
S_5	Good	(0.667, 0.833, 1)
S_6	Very good	(0.833, 1, 1)

shows nine-linguistic variables with triangular fuzzy membership function (Jadhav and Bajaj, 2010).

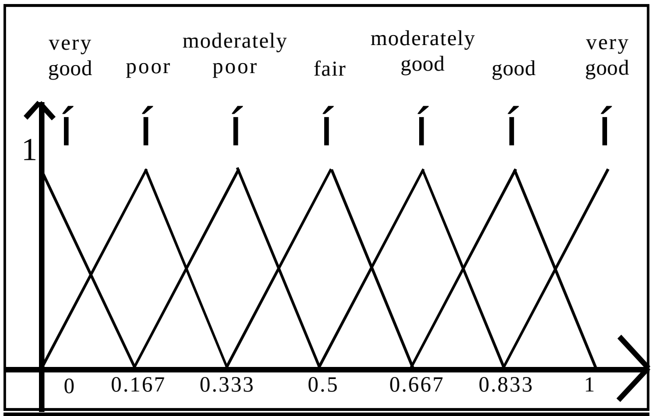

Fig. 2: Seven-Linguistic variables with Triangular Fuzzy Membership Function

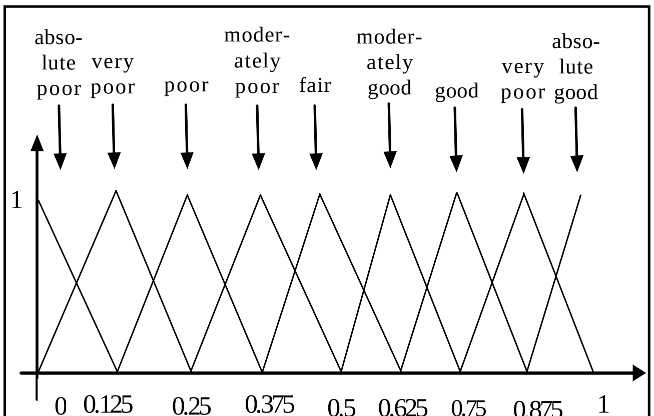

Fig. 3: Nine-Linguistic variables with Triangular Fuzzy Membership Function

Operation of triangular fuzzy number :

Let $a = (a^l, a^m, a^u)$ and $b = (b^l, b^m, b^u)$ be two Triangular Fuzzy Numbers (TFNs) and is a positive real number, two important operations are used in this paper as follows :

$$a = (a^l, a^m, a^u) \quad b = (b^l, b^m, b^u) = (a^l + b^l, a^m, b^m, a^u, b^u)$$

$$\alpha \cdot (a^l, a^m, a^u) = (\alpha\alpha^l, \alpha a^m, \alpha a^u)$$

Normalization of Triangular Fuzzy Number (NTFN) :

Let matrix

$A = [a_{ij}]_{k \times n}$,

which,

$a_{ij} = (a_{ij}^l, a_{ij}^m, a_{ij}^u)$ is the Triangular Fuzzy Number that being normalized, and result in matrix

$B = [b_{ij}]_{k \times n}$

which,

$b_{ij} = (b_{ij}^l, b_{ij}^m, b_{ij}^u)$

as follows :

$$\begin{cases} b_{ij}^l = a_{ij}^l \,/\, \sqrt{\sum_{i=1}^{k} \left(a_{ij}^u\right)^2} \\ b_{ij}^m = a_{ij}^m \,/\, \sqrt{\sum_{i=1}^{k} \left(a_{ij}^m\right)^2} \\ b_{ij}^u = a_{ij}^u \,/\, \sqrt{\sum_{i=1}^{k} \left(a_{ij}^l\right)^2} \end{cases} \qquad \text{........(2)}$$

Fuzzy topsis : an overview :

The TOPSIS approach is a MCDM method, developed by (Chang, 1992; Chen, 2002 and Jadhav and Bajaj, 2013); and many other researchers have been working in this field. Using the TOPSIS method, the best alternative must have the shortest distance to the Positive Ideal Solution (PIS) and the longest distance to the Negative Ideal Solution (NIS) (Chang, 1992; Chen, 2002).

Suppose that a decision making problem have *k* evaluation alternatives $A = (a_1, a_2, ..., a_k)$, n evaluation criteria $C = (c_1, c_2, ..., c_n)$, priority vector of criteria $w = (w_1, w_2, ..., w_n)$, and the evaluation matrix :

$X = [x_{ij}]_{k \times n}$ as follows :

Table 2 : Converting nine-linguistic expressions to triangular fuzzy numbers

Fuzzy label	Fuzzy linguistic expression	TFNs
S_0	Absolute poor	(0, 0, 0.125)
S_1	Very poor	(0, 0, 0.125, 0.25)
S_2	Poor	(0.125, 0.25, 0.375)
S_3	Moderately poor	(0.25, 0.375, 0.5)
S_4	Fair	(0.375,0.5,0.625)
S_5	Moderately good	(0.5,0.625,0.75)
S_6	Good	(0.625,0.75,0.875)
S_7	Very good	(0.75,0.875,1)
S_8	Absolute good	(0.875,1,1)

$$x = \begin{array}{c|cccc} & c_1 & c_2 & c_j & c_n \\ a_1 & x_{11} & x_{12} & x_{1j} & x_{1n} \\ a_2 & x_{21} & x_{22} & x_{2j} & x_{2n} \\ a_i & x_{i1} & x_{i2} & x_{ij} & x_{in} \\ a_k & x_{k1} & x_{k2} & x_{kj} & x_{kn} \end{array}$$

X_{ij} expresses evaluation valve of alternative a_i respective to criterion c_j. x_{ij} is presented in Linguistic Variable and Triangular Fuzzy Number (TFNs)

$$X_{ij} = (x_{ij}^{l}, x_{ij}^{m}, x_{ij}^{u})$$

Fuzzy Ideal Solution (FIS) :

The 'Fuzzy Positive Ideal Solution' (FPIS) which has '*the best evaluation value*' respective to each criterion is determined as follows (Jadhav and Bajaj, 2010) :

$$A^{+} = \left[X_1^{+}, X_2^{+}, ..., X_n^{+}\right] \text{ where, } X_j^{+} = \max_{i=l,...k} (X_{ij})$$

$$= \left(\max_{i=l,...k} (X_{ij}^{l}), \max_{i=l,...k} (X_{ij}^{m}), \max_{i=l,...k} (X_{ij}^{u})\right) \quad j = 1,...,n \tag{3}$$

The 'Fuzzy Negative Ideal Solution' (FNIS) which has '*the worst evaluation value*' respective to each criterion is determined as follows (Jadhav and Bajaj, 2010) :

$$A^{-} = \left[X_1^{-}, X_2^{-}, ..., X_n^{-}\right] \text{ where, } X_j^{-} = \max_{i=l,...k} (X_{ij})$$

$$= \left(\min_{i=l,...k} (X_{ij}^{l}), \min_{i=l,...k} (X_{ij}^{m}), \min_{i=l,...k} (X_{ij}^{u})\right) \quad j = 1,...,n \tag{4}$$

Distance to fuzzy ideal solution :

Let $a = (a^l, a^m, a^u)$ and $b = (b^l, b^m, b^u)$ be two triangular fuzzy numbers. The distance between a and b can be calculated by using the vertex method (Jadhav and Bodakhe, 2013).

$$d(a,b) \sqrt{\frac{1}{3}\left[(a^l - b^l)^2\right] + \left[(a^m - b^m)^2\right] + \left[(a^u - b^u)^2\right]} \tag{5}$$

Then, the distance between each alternative to FPIS and FNIS can be, respectively derived from :

$$d_i^{+} = \sum_{j}^{n} d(x_{ij}, x_j^{+}) \quad i = 1, 2, ..., k$$

$$d_i^{-} = \sum_{j}^{n} d(x_{ij}, x_j^{-}) \quad i = 1, 2, ..., k \tag{6}$$

Closeness co-efficient :

Closeness co-efficient R_i of each alternative is used to determine the ranking of all alternatives. The higher value of closeness co-efficient indicates that corresponding alternative is closer to FPIS and farther from FNIS simultaneously (Chang, 1992).

$$R_i = \frac{d_j^{-}}{d_j^{-} + d_j^{+}} \quad i = 1, 2, ..., k \tag{7}$$

Basic steps of fuzzy topsis method :

Chen, Chu, Saghafian *et al.* (Jadhav and Bodakhe, 2013; Saghafian and Hejazi, 2005) and other researchers have expanded the traditional TOPSIS method into the Fuzzy TOPSIS method in order to handle fuzziness in decision making problem. This paper proposes a modified Fuzzy TOPSIS method to deal with triangular fuzzy number (TFN) with modification of linguistic variable, TFN normalization and distance to ideal solution. Basic step of this Fuzzy TOPSIS method can be described follows:

– Obtain fuzzy evaluation matrix $X = [x_{ij}]_{k \times n}$ for k alternatives over n criteria. Preference data is expressed first in linguistic variable, and then converted to TFN.
– Normalize fuzzy evaluation matrix X by equation (2).
– Multiply the priority vector of the criteria with the normalized evaluation fuzzy matrix resulting in matrix $Y = [y_{ij}]_{k \times n}$ with $y_{ij}*w_j$.
– Identify the Fuzzy Positive Ideal Solution (FPIS) A+ and Fuzzy Negative Ideal Solution (FNIS) A^- of matrix Y referring to equations (3) and (4).
– Calculate fuzzy distance d_i^+ and d_i^- over each alternative to FPIS and FNIS, respectively referring to equations (5) and (6).
– Determine the closeness co-efficient R_i referring to equation (7) for each alternative.
– Rank order of alternatives by maximizing closeness co-efficient R_i.

Ranking alternatives :

After the priority vector of criteria is determined by the Fuzzy TOPSIS method is used to rank the alternatives. Linguistic variables are applied to obtain the important preference of each alternative respective to each criterion. As a result, the evaluation matrix is formed. This step was

Table 3: Linguistic preferences of alternatives respective to each criterion

	C_1	C_2	C_3	C_4
A_1	Good	Moderately poor	Fair	Good
A_2	Poor	Very good	Good	Fair
A_3	Very good	Moderately good	Poor	Moderately poor

illustrated by applying the procedure presented in section 3.

Numerical example :

Suppose that someone wants to find a location to open a restaurant and there are three potential restaurant's locations. In order to select an appropriate location, there are four criteria to consider: population base, parking area, accessibility and visibility. The hierarchical structure of decision making problem is formed as shown in Fig. 4.

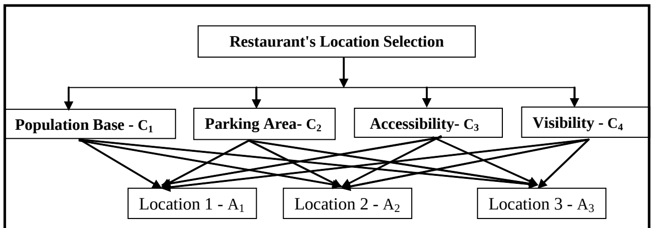

Fig. 4: Hierarchical structure of restaurant's location decision making

Applying fuzzy TOPSIS method in section 3, ranking of alternatives will be determined. Decision maker uses nine-linguistic expressions to express the preference of alternatives respective to each criterion as shown in Table 3.

Referring to Table 2, linguistic preferences are converted to fuzzy number as shown in Table 4:

Applying equation (2), the normalized fuzzy decision matrix is formed as shown in Table 5:

Multiply priority vector of criteria with normalized Fuzzy matrix which is shown in Table 6:

Using equations (3) and (4), *i.e.* Fuzzy Positive Ideal Solution (FPIS) and Fuzzy Negative Ideal Solution (FPIS) are calculated, respectively as follows :

FPIS A^+ = [(0.12,0.164,0.224), (0.098,0.136,0.353), (0.154,0.225,0.333), (0.169,0.247,0.365)]

FPIS A^- = [(0.02,0.047,0.084), (0.033, 0.058, 0.177), (0.031,0.075, 0.143),(0.068,0.123,0.208)]

Referring to equations (5) and (6), the distances from each alternative to FPIS and FNIS, respectively, are calculated as follows :

	A_1	A_2	A_3
d^+	0.220	0.206	0.345
d^-	0.304	0.318	0.179

Finally, using equation (7), the closeness co-efficients are calculated as follows :

$$R_1 = \frac{d_1^-}{d_1^- + d_1^+} = 0.582 \qquad R_2 = \frac{d_2^-}{d_2^- + d_2^+} = 0.607$$

$$R_3 = \frac{d_3^-}{d_3^- + d_3^+} = 0.342$$

According to the closeness co-efficient of the three alternatives, the order of the three alternatives is $A_2 > A_1 > A_3$. Location 2 (A_2) would be selected for opening the restaurant. Similar work related to the present work was also done by Jadhav and Bajaj (2011); Jadhav and Bajaj (2012); Wang and Elhag (2006) and Zadeh (1975).

Concluding remarks :

In this paper, we propose a new MCDM verses Fuzzy TOPSIS method to handle the decision making problems in a fuzzy environment, where the information is uncertain and vague. The uncertain and vague preferences are first presented in linguistic variables and then converted to triangular fuzzy

Table 4 : Fuzzy number preference of alternatives respective to each criterion

	C_1	C_2	C_3	C_4
A_1	(0.625, 0.75, 0.875)	(0.25, 0.375, 0.5)	(0.375, 0.5, 0.625)	(0.625, 0.75, 0.875)
A_2	(0.125,0.25,0.375)	(0.75, 0.875,1)	(0.625, 0.75, 0.875)	(0.375,0.5,0.625)
A_3	(0.75,0.875,1)	(0.5,0.625,0.75)	(0.125,0.25,0.375)	(0.25,0.375,0.5)

Table 5 : Normalized fuzzy number preference

	C_1 (0.2209)	C_2 (0.1767)	C_3 (0.2811)	C_4 (0.3213)
A_1	(0.453, 0.636, 0.889)	(0.19, 0.329, 1)	(0.329, 0.53, 0.845)	(0.527, 0.76, 1.136)
A_2	(0.091,0.212,0.381)	(0.56, 0.768,2)	(0.549, 0.8, 1.183)	(0.316,0.51,0.811)
A_3	(0.543,0.742,0.016)	(0.37,0.549,1.5)	(0.11,0.27,0.507)	(0.211,0.38,0.649)

Table 6 : Weighted normalized fuzzy number preference

	C_1	C_2	C_3	C_4
A_1	(0.1, 0.14, 0.196)	(0.033, 0.058, 0.177)	(0.093, 0.15, 0.238)	(0.169, 0.247, 0.365)
A_2	(0.02,0.047,0.084)	(0.098, 0.136, 0.353)	(0.154, 0.225, 0.333)	(0.102,0.165,0. 261)
A_3	(0.12,0.164,0.224	(0.066,0.097,0.265)	(0.031,0.075, 0.143)	(0.068,0.123,0.208)

numbers. The problem with calculating priority vector of criteria is presented as an optimization problem and it is solved by using TOPSIS method. To find the priority vector, which maximizes triangular membership function. After determining the priority vector of criteria, the proposed method is used to evaluate rank of the order of alternatives. From the TOPSIS method the best ideal solution out of three locations is A_2. A_1 is second choice for selection of locations. The decision maker will be choosing the second (restaurant location) alternative according to Fuzzy TOPSIS method, we get same ranking preferences. Then the decision maker chooses the best alternative *i.e.* A_2.

TOPSIS method utilizes the advantages of fuzzy set Theory, therefore, the decision making becomes realistic and effective. By using proposed method decision maker may choose best "optimal" (most favorable) alternative selection everywhere. It has large applicability in real-life situations such as the selection of T.V., Mobile, Laptop, and so on. The proposed fuzzy TOPSIS method considers the decision makers (DMs) preference that is an advantages of it. Moreover, it seems that the proposed method flexible and easy to use.

It is expected that the fuzzy TOPSIS method have more potential applications in the near future. A numerical example of selecting restaurant's location is also presented to clarify the procedure of the proposed method.

REFERENCES

Chang, D.Y. (1992). Extent analysis and synthetic decision, optimization techniques and applications. *World Scientific, Singapore,* **1**: 352-355.

Chen, C.T. (2002). Extensions of the TOPSIS for group decision making under fuzzy environment. *Fuzzy Sets & Systems,* **114** (16) : 1-9.

Chu, T.C. (2002). Selecting plant location via a fuzzy TOPSIS approach. *Internat. J. Adv. Manufact. Tech.,* **20** (11) : 859-864.

Hwang, C.L. and Yoon, K. (1981). Multiple attribute decision making: method and applications, *Berlin: Springer*.

Jadhav, Omprakash and Bajaj, V.H. (2010). Multi-objective inventory model of deteriorating items with fuzzy inventory Cost and some fuzzy constraints. *Internat. J. Agric. & Stat. Sci.,* **6**(2) : 529-538.

Jadhav, Omprakash and Bajaj, V.H. (2013). Multi-objective inventory model of deteriorating items with shortages in fuzzy environment. *Internat. J. Statistika & Mathematika,* **6**(1): 2277- 2790.

Jadhav, Omprakash and Bodakhe, S.D. (2013). Additive fuzzy multiple goal programming model for the unbalanced transportation problem with hyperbolic membership function. *Internat. J. Statistika & Mathematika,* **6**(3) : 105-108.

Jadhav, V.S. and Bajaj, V.H. (2011). Application of fuzzy TOPSIS method for solving job-shop scheduling problem. *Internat. J. Operations Res. & Optimization,* **2**(2) : 333-342.

Jadhav, V.S. and Bajaj, V.H. (2012). Selection of job shop scheduling problem using fuzzy linguistic variables. *Internat. J. Statistika & Mathematika,* **3** (1) : 1-5.

Lai, Y.J., Liu, T.Y. and Hwang, C.L. (1994). TOPSIS for MODM. *European J. Operational Res.,* **76** (3) : 486-500.

Liu, W. and Liu, P. (2010). Hybrid multiple attribute decision making method based on relative approach degree of grey relation projection. *J. Busi. Mgmt.,* **4**(17) : 3716-3724.

Mikhailov, L. and Tsvetinov, P. (2004). Evaluation of services using a fuzzy analytic hierarchy process. *Appl. Soft Comput.,* **5** (1) : 23-33.

Saghafian, S. and Hejazi, S.R. (2005). Multi-criteria group decision making using a modified fuzzy TOPSIS procedure. *Web Technologies & Internet Comm.,* **2** : 215-221.

Wang, Y.M. and Elhag, T.M.S. (2006). Fuzzy TOPSIS method based on alpha level sets with an applications to bridge risk assessment. *Expert System with Applications,* **31** (2) : 309-319.

Zadeh, L.A. (1975). The concept of a linguistic variable and its application to approximate reasoning, Part I. *Informa. Sci.,* **8**(3) : 199–249.

WEBLIOGRAPHY

Amarender, R. and Devraj, M. (2010). Growth and instability in chickpea production in India. *www.krisat.org Accessed on 15 February 2011*.