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RESEARCH PAPER

Bayesian analysis of simple linear econometric model on the natural parameter space

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ABSTRACT

In this paper we have considered simple linear econometric model (SLEM) on the natural parameter space. The unknown parameter β have been estimated in a Bayesian frame work assuming the squared error loss function and a suitable prior density on the parameter space.

Key Words : Bayesian analysis, Squared error loss function, Likelihood function, Bayes estimator

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In many problems of economics and econometrics, we often encounter sets of data which can be handled by the analysis of general linear econometric model [GLEM]. For example Okun's Law which relates GDP growth to the unemployment rate. Many authors have carried out the Bayesian statistical analysis of such a model. The fundamental work of Zellner (1971) is worth mentioning here. Diff erent authors have used different prior densities for the underlying parameters covering the cases of non- informative and informative priors including the natural conjugate densities. Here we mentioned those of Raiffa and Schlaifer (1961), Box and Tiao (1973).

Some papers on Bayesian Econometric analysis using non-conjugate priors were published by Bhattacharya and Lal (1985), Bhattacharya and Saxena (1986), Bhattacharya and Lal (1990). In one of the papers of Bhattacharya and Lal, the underlying parameters were assumed to take values in a restricted domain.

Here we present the Bayesian analysis of simple linear econometric model (SLEM) on the natural parameter space that is slope involved in the model which is known in advance to take values from - ∞ to + ∞. Let us postulate SLEM specified by the equation

$$\underset{\sim}{\mathbf{y}} = \underset{\sim}{\mathbf{x}}\beta + \underset{\sim}{\mathbf{u}} \qquad (1)$$

where

$\underset{\sim}{y}' = (y_1, y_2 \ldots\ldots\ldots\ldots\ldots\ldots y_n)$ is a point in R^n and represents observations on an endogenous variable, $\underset{\sim}{x}' = (x_1, x_2 \ldots\ldots\ldots\ldots\ldots\ldots x_n)$ is a vector of observations on an exogenous variable, β is an unknown slope and $\underset{\sim}{u}' = (u_1, u_2 \ldots\ldots\ldots\ldots\ldots\ldots u_n)$ is a vector of independent identically distributed normal random disturbances with zero means and common variance σ^2.

In the present work, the Bayesian analysis is carried out by assuming suitable prior density on the parameter space of (β, σ) and the squared error loss function (SELF). The prior probability density function is assumed to be normal, where mean of the normal prior and *a*' are known on the basis of prior knowledge.

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$$g(\beta/\sigma) \propto \frac{1}{\sigma} e^{\frac{a'}{2\sigma^2}(\beta-\bar{\beta})^2}, (-\infty < \bar{\beta} < \infty, a' > 0) \qquad (2)$$

The prior density of disturbance variance is assumed $\sigma^2 \sim IG(m, \alpha)$, the inverse Gaussian density (cf. Johnson and Kotz, 1970 p. 137). The parameters m and α are assumed to be known on the basis of the prior knowledge.

$$g(w) = \alpha^{\frac{1}{2}} (2\pi)^{\frac{1}{2}} w^{\frac{3}{2}} e^{\frac{\alpha}{2m^2 w}(w-m)^2} \quad (0<w<\tilde{a};\, m>0;\, \sim>0) \qquad (3)$$

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Since the prior probability density function of σ follows (4) so that the joint prior probability density function on the parameter of (β, σ) is given in (5).

$$g(\sigma) \propto \frac{1}{\sigma^3} e^{\frac{\alpha}{2m^2\sigma^2}(\sigma^2-m)^2}, (-0 < \sigma < \infty) \qquad (4)$$

$$g(\beta,\sigma) \propto \frac{1}{\sigma^4} e^{\frac{1}{2\sigma^2}\left[\alpha'(\beta-\bar{\beta})^2 + \frac{\alpha}{m^2}(\sigma^2-m^2)\right]}, (-\infty < \beta < \infty, 0 < \sigma > \infty) \qquad (5)$$

The likelihood function

$$\ell(\beta,\sigma) \propto \sigma^{-n} e^{\frac{1}{2\sigma^2}\sum_{i=1}^{n}(y_i-\beta x_i)^2}, [\beta \in R; \sigma \in (0,\infty)] \qquad (6)$$

The joint posterior density of (β, σ) via Bayes theorem is as follows:

$$g^*(\beta,\sigma) \propto \frac{1}{\sigma^{n+4}} e^{\frac{1}{2\sigma^2}\left[a'(\beta-\bar{\beta})^2 + \frac{\alpha}{m^2}(\sigma^2-m)^2 + \sum_{i=1}^{n}(y_i-\beta x_i)^2\right]}, (-\infty < \beta < \infty; 0 < \sigma < \infty) \qquad (7)$$

$$g^*(\beta,\sigma) \propto \frac{1}{\sigma^{n+4}} e^{\frac{Q(\beta)}{2\alpha^2} - \frac{\alpha}{2m^2}\alpha^2} \qquad (8)$$

$$Q(\beta) = \left(a' + \sum_{i=1}^{n} x_i^2\right)\left[\left(\beta - \frac{a'\bar{\beta} + \sum_{i=1}^{n} x_i y_i}{a' + \sum_{i=1}^{n} x_i^2}\right)^2 + \frac{1}{\left(a' + \sum_{i=1}^{n} x_i^2\right)^2}\left\{\left(a'\bar{\beta} + \sum_{i=1}^{n} y_i^2\right)\left(a' + \sum_{i=1}^{n} x_i^2\right) - \left(a'\bar{\beta} + \sum_{i=1}^{n} x_i y_i\right)^2 + \alpha\left(a' + \sum_{i=1}^{n} x_i^2\right)\right\}\right]$$

$$= \left(a' + \sum_{i=1}^{n} x_i^2\right)\left[\left(\beta - \frac{a'\bar{\beta} + \sum_{i=1}^{n} x_i y_i}{a' + \sum_{i=1}^{n} x_i^2}\right)^2 + \frac{1}{\left(a' + \sum_{i=1}^{n} x_i^2\right)^2}\left\{a'\sum_{i=1}^{n}(y_i - \beta x_i)^2 + \left(\sum_{i=1}^{n} x_i^2\right)\left(\sum_{i=1}^{n} y_i^2\right) - \left(\sum_{i=1}^{n} x_i y_i\right)^2 + \alpha\left(\alpha' + \sum_{i=1}^{n} x_i^2\right)\right\}\right] \qquad (9)$$

From (9) we have (10), (11) and (12)

$$Q(\beta) = \left(a' + \sum_{i=1}^{n} x_i^2\right)\left[(\beta - d_0)^2 + b_0\right] \qquad (10)$$

$$d_0 = \frac{a'\bar{\beta} + \sum_{i=1}^{n} x_i y_i}{a' + \sum_{i=1}^{n} x_i^2} \qquad (11)$$

$$b_0 = \frac{1}{\left(a' + \sum_{i=1}^{n} x_i^2\right)^2}\left[a'\sum_{i=1}^{n}(y_i - \beta x_i)^2 + \left(\sum_{i=1}^{n} x_i^2\right)\left(\sum_{i=1}^{n} y_i^2\right) - \left(\sum_{i=1}^{n} x_i y_i\right)^2 + \alpha\left(a' + \sum_{i=1}^{n} x_i^2\right)\right] \qquad (12)$$

The marginal posterior density

$$g^*(\beta) \propto \int_0^{\infty} t^{\frac{(n+4)}{2}} e^{\frac{Q(\beta)}{2t} - \frac{\alpha}{2m^2}t}\, dx \qquad (13)$$

The above integral is evaluated in terms of the modified Bessel function of third kind K_v (t) using its integral representation (cf. Gradshteyn and Ryzhik, 1965, p. 340) we obtain the Bayesian posterior density of β.

$$\int_0^{\infty} x^{v-1} e^{\frac{-b}{x} - cx}\, dx = 2\left(\frac{b}{c}\right)^{\frac{v}{2}} K_v(2\sqrt{bc})\; b,c > 0 \qquad \left| \begin{array}{l} v = -\frac{(n+2)}{2} \\ b = -\frac{Q(\beta)}{2} \\ c = -\frac{\alpha}{2m^2} \end{array} \right. \qquad (14)$$

$$g^*(\beta) \propto 2\left(\frac{m^2 Q(\beta)}{\alpha}\right)^{\frac{-(n+2)}{4}} K_{\frac{-(n+2)}{2}}\left(\sqrt{\frac{\alpha}{m^2}Q(\beta)}\right) (-\infty < \beta < \infty) \qquad (15)$$

$$g^*(\beta)\, CK_{\frac{-(n+2)}{2}}\left(\sqrt{\frac{\alpha}{m^2}\left(a' + \sum_{i=1}^{n} x_i^2\right)\left[b_0 + (\beta - d_0)^2\right]}\right) 2\left(\frac{m^2}{\alpha}\left(b_0 + (\beta - d_0)^2\right)\right)^{\frac{-(n+2)}{4}} \qquad (16)$$

Since $g^*(\beta)$ is a normalized posterior density function, so we have:

$$\int_{-\infty}^{\infty} CK_{\frac{-(n+2)}{2}}\left(\sqrt{\frac{\alpha}{m^2}\left(a' + \sum_{i=1}^{n} x_i^2\right)\left[b_0 + (\beta - d_0)^2\right]}\right) 2\left(\frac{m^2}{\alpha}\left(b_0 + (\beta - d_0)^2\right)\right)^{\frac{-(n+2)}{4}} d\beta = 1 \qquad (17)$$

Under the assumptions of the squared error loss function, the Bayes estimator of b is simply the posterior expectation of b obtained from $g^*(\beta)$ at (16).

$$E(\beta) = \int_{-\infty}^{\infty} \beta g^*(\beta)\, d\beta \qquad (18)$$

$$E(\beta) = \frac{a'\bar{\beta} + \sum_{i=1}^{n} x_i y_i}{a' + \sum_{i=1}^{n} x_i^2} \qquad (19)$$

RESULTS AND REMONSTRATION

In this section my aim is to present the performance of $E(\beta)$ on the Data set of Ministry of Agriculture, Government of India (2005) on the area (in million ha) and yield (in kg/ha) of total pulses of year 1950 to 2004.

Table 1 is a tabulation of the estimated β, where it is slightly fallen with increase in known parameter a', which is shown in Fig. 2. If we assume that the precision of the prior distribution tends to zero, that is, $m \to 0$ then Bayes estimate

Table 1: Performance of $E(\ ) = \hat{\ }$ with the different values of a'

a'	$E(\beta) = \hat{\beta}$	a'	$E(\beta) = \hat{\beta}$
1	24.54629	48	24.53806
2	24.54611	57	24.53649
3	24.54594	69	24.5344
4	24.54576	79	24.53266
5	24.54558	92	24.53041
9	24.54488	105	24.52816
14	24.544	146	24.52111
26	24.5419	203	24.51139
37	24.53998		

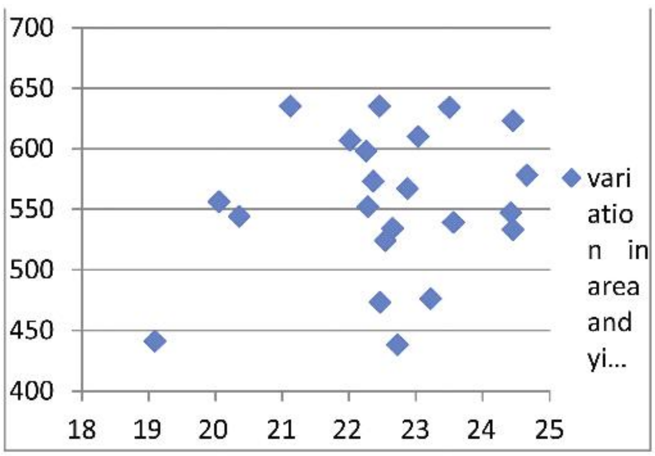

Fig. 1 : **Area and yield of total pulses**

tends to ordinary least square estimate (OLSE).

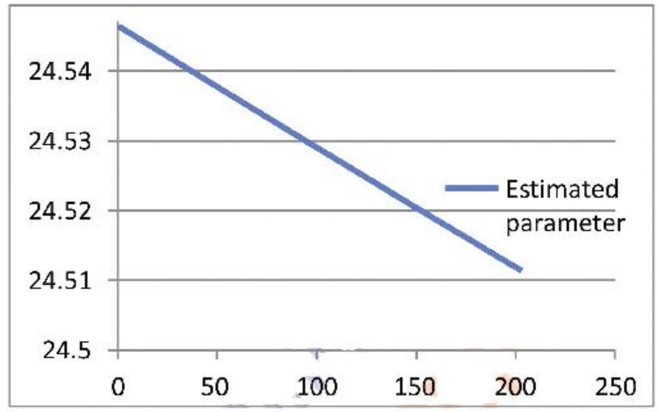

Fig. 2 : **Performance of $\hat{\ }$**

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