RESEARCH PAPER International Journal of Agricultural Engineering | Volume 8 | Issue 1 | April, 2015 | 60–65
e ISSN–0976–7223 ■ Visit us : *www.researchjournal.co.in* ■ **DOI: 10.15740/HAS/IJAE/8.1/60-65**

Comparison between two different conceptual mathematical models in prediction of direct runoff hydrographs from a small watershed

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Received : 23.12.2014; **Revised :** 19.02.2015; **Accepted :** 03.03.2015

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■ **ABSTRACT :** In the present study, two mathematical models namely (i) Lag and route model and (ii) Muskingum model which are based on unit-step and transfer functions approach were developed for runoff prediction from Shenda park watershed treating the watershed as lumped, linear and time-invariant system. The hydrological data of the study watershed were collected from zonal station of National Agricultural Research Project, Shenda Park, Kolhapur (M.S.) for the years 2000 to 2008. Out of twelve single storm events, nine storm events were included in the analysis for parameters estimation and remaining three storm events were considered for prediction purposes. The model parameters, *viz.*, lag time and (τ) and storage co-efficient (K) of Lag and route model were estimated by the methods of cumulants (Singh, 1988) and moments (Nash, 1957) whereas the model parameter storage constant (K) for Muskingum model was estimated by using method suggested by Jawed (1973). Performance evaluation of these two developed model in determining direct runoff hydrograph ordinates were evaluated using various statistical indices such as correlation co-efficient (R), special correlation co-efficient (Rs.), co-efficient of efficiency (CE) and root mean square error (RMSE). The results showed that both the developed model can be used for prediction of the direct run off hydrograph from the study watershed, however, direct runoff hydrographs obtained through Muskingun models are much closer to actual observed direct runoff hydrograph than that of Lag and route model.

■ **KEY WORDS :** Lag and route model, Muskingum model, Unit-step transfer functions

■ **HOW TO CITE THIS PAPER :** Bansude, S.N., Chunale, G.L. Shinde, A.A. and Kumar, Pravendra (2015). Comparison between two different conceptual mathematical models in prediction of direct runoff hydrographs from a small watershed. *Internat. J. Agric. Engg.*, **8**(1) : 60-65.

Rainfall-runoff process is an extremely complex and difficult problem involving many variables, which are interconnected in many complicated way. The problem of estimating runoff from small watersheds is an important element in the design of hydraulic structure, such as storm sewers, spillway, highway drainage, diversion works, bridges, culverts and flood retarding structures. Although the cost of each hydraulic structure, constructed in small watersheds may be low but projects constructed in large number represented the considerable national expenditure. Over design of hydraulic structures could represent the considerable waste of money. Likewise under-estimation of runoff would cause large-scale damage which would be equally wasteful. Therefore, studies on generation of future events with regard to stream flow modeling are likely to play an important role in proper planning and management of available water resources.

Estimation of flood hydrographs can be achieved by different methods and hydrological models are one of them. The transformation of rainfall to runoff is a complex physical phenomenon, which is yet to be fully understood. In the hydrology of natural catchments, rainfall-runoff relations are usually non-linear. However, the linear theory is frequently adopted because it is mathematically much easier to handle than the better fitting non linear models. Therefore, assumption of linearity and time-invariant has been considered a convenient starting point for handling input-output relationships in hydrologic study. Ideally, a conceptual model based on sound physical principles would be the best approach to predict runoff from a given rainfall. Among the many hydrologic models used for flood routing in natural channels and reservoirs, the Lag and route and Muskingum models are most frequently used tools, because of their simplicity and involvement of fewer parameters.

In the present study, conceptual Lag and route model and Muskingum model using unit-step and transfer function approach as suggested by Wang and Wu (1983) were developed for determining direct runoff hydrographs for small watershed of 12 ha area developed at National Agricultural Research Project (NARP), Shenda Park, Kolhapur of Maharashtra state, India, considering watershed as a lumped, linear and time-invariant system.

■ METHODOLOGY

A small leaf shaped watershed developed at NARP, Shenda Park, Kolhapur (Maharashtra) located at 16°45' N latitude and 74°14' E longitude having area of 12 ha was selected for the present study. The rainfall and runoff data of Shenda park watershed were collected for the years 2000 to 2008 from zonal station of National Agricultural Research Project, Shenda Park, Kolhapur (M.S.). In this study, twelve single peaked and isolated storm events were selected. A calibration set containing nine events and verification sets consisted of three storm events which were used for estimating model parameters and for prediction purpose. The rainfall data recorded at zonal station of National Agricultural Research Project, Shenda Park, Kolhapur (M.S.) were used for the derivation of mass rainfall curves as suggested by Subramanya (2003). In this study the φ-index method was used for the estimation of the average infiltration rate and from which effective rainfall for respective storm events were estimated. The selected stage hydrographs were converted into discharge hydrographs using the relationship reported by Wasiullah *et al.* (1972). The values of ordinates of direct runoff hydrographs for all the twelve storm events were estimated by subtracting base flow ordinates from the total discharge hydrographs. The method suggested by Chow (1964) was used for separating the base flow from the total runoff hydrograph. The analysis of the data were done by using standard software Microsoft Excel.

Development of conceptual mathematical models:

Application of system analysis in hydrology has brought about one of the greatest advances in modern hydrological technology. Generally speaking, a system consists of an input, an output and the transformation whereby the input is transformed into the output. In the hydrological context, a basin is considered as the system in which an input of effective rainfall is transformed in to an output as discharge at the outlet.

The input output relationship of a linear time-invariant system of a basin can be represented by the linear differential equation reported by (Ogata, 1970) is given as :

$$a_n \frac{d^n Q(t)}{dt^n} + a_{n-1} \frac{d^{n-1} Q(t)}{dt^{n-1}} + \dots + a_1 \frac{Q(t)}{dt} + a_0 Q(t) = b_m \frac{d^m 1(t)}{dt^m} + b_{m-1} \frac{d^{m-1} I(t)}{dt^{m-1}} + \dots + b_1 \frac{1(t)}{dt} + b_0 I(t) \quad ....(1)$$

in which, Q and I are the output (*i.e.* the outflow) and the input (*i.e.* the effective rainfall), respectively and $a_n, a_{n-1}, \dots a_1, a_0; b_m, b_{m-1}, \dots b_1, b_0$ are the positive integers with n > m (Kulandaiswamy and Babu, 1975). The detailed analytical derivation of the model has been described by Kumar *et al.* (2008).

In the analysis or study of a system an appropriate model must be selected. In this study, two conceptual models namely (i) Lag and route model and (ii) Muskingum model (reservoir routing *i.e.* X=0.00) were developed for determining direct runoff hydrographs from the study watershed.

Derivation of outflow for runoff prediction models:

The detailed analytical derivation of the Lag and route model as well as Muskingum model (Reservoir routing approach) has been described by Kumar *et al.*

(2008).

The expression of Lag and route model for direct runoff prediction developed as per procedure suggested by Kumar *et al.* (2008) is given below :

$$Q(t) = \sum_{i=0}^{m} (P_{i+1} - P_i)\left[1 - e^{-\frac{(t - \tau - i\Delta t)}{K}}\right] u(t - \tau - \Delta t) : P_0 = P_{m+1} = 0 \quad ..(2)$$

where, Q(t) is the ordinate of the direct runoff hydrographs at time t, P_i is the i^{th} effective rainfall, Δt is the time interval and m is total number of rainfall blocks. The parameters, *viz.*, lag time (τ) and storage co-efficient (K) of the model were estimated by the methods of cummulants (Singh, 1988) and Moments (Nash, 1957) using rainfall and runoff data of watershed of nine storm events used calibration. The estimated parameters are shown in the Table 1. The estimated average values of the parameters *viz.*, lag time τ, equal to 0.1572 and storage co-efficient K, equale to 0.3515 were substituted in equation (2) to develop Lag and route model for computing the direct runoff hydrograph ordinates can be expressed as :

$$Q(t) = \sum_{i=0}^{m} (P_{i+1} - P_i)\left[1 - e^{-\frac{(t - 0.1572 - i\Delta t)}{0.3513}}\right] u(t - 0.1572 - \Delta t) : P_0 = P_{m+1} = 0 \quad .....(3)$$

The expression of Lag and route model for direct runoff prediction developed as per procedure suggested by Kumar *et al.* (2008) is given below :

$$Q(t) = \sum_{i=0}^{m} (P_{i+1} - P_i)\left[1 - \frac{1}{1 - X} e^{-\frac{(t - i\Delta t)}{K(1 - X)}}\right] u(t - i\Delta t) P_0 = 0, P_{m+1} = 0 \quad (4)$$

where, Q(t) is the ordinate of the direct runoff hydrograph at time t, P_i is i^{th} effective rainfall, Δt is time interval and m is total number of rainfall blocks.

For reservoir routing, the value of the parameter X is taken as zero. The value of model parameter, storage constant K, is determined considering the discharge at the time of the maximum slope on the recession of the semi-log hydrograph of the recession curve based relationship as suggested by Jawed (1973) :

$$K = -\frac{Q_t}{\frac{\Delta Q}{\Delta T}} \quad .....(5)$$

where, Q_i is the discharge at point of inflection, $\Delta Q/\Delta T$ is the slope of the straight line passing through point of inflection, and ΔQ is the incremental runoff rate for incremental time ΔT. The estimated values of storage time constant for nine storm event (calibrated event) are given in the Table 1.

By substituting the average value of parameter X=0.00 and storage time constant (K) = 0.37 (hr) in the equation (5), the final expression for Muskingum model for prediction of direct runoff hydrographs from study watershed is given as :

$$Q(t) = \sum_{i=0}^{m} (P_{i+1} - P_i)\left[1 - e^{-\frac{(t - i\Delta t)}{0.37}}\right] u(t - i\Delta t) : P_0 = 0, P_{m+1} = 0 \quad .....(6)$$

Performance evaluation of model :

To evaluate the model, five statistical parameters *viz.*, correlation co-efficient (R) (Sarma *et al.*, 1973), special correlation co-efficient (R) (Eagleson and March, 1965), co-efficient of efficiency (CE) (Nash and Sutcliffe, 1970) and root mean square error (RMSE) (Yu *et al.*, 1994) were used for the purpose. The detailed procedure and formulae for these statistical indices are given by Kumar *et al.* (2008).

RESULTS AND DISCUSSION

Out of the twelve storm events selected in the study, nine storm events were used to calibrate the model while three storm events were used for the validation purpose of two models selected for the comparison. The performance of these two models was tested by comparing observed and predicted direct runoff hydrographs for the calibration and verification sets. Regenerated runoff hydrographs for representative calibration event of October 9, 2000 and one predicted event of June 15, 2004 are shown in (Fig. 1 and 2), respectively and for both the cases good approximations to the actual runoff hydrographs were noted. It is clear from the figures that the computation of peak runoff rate

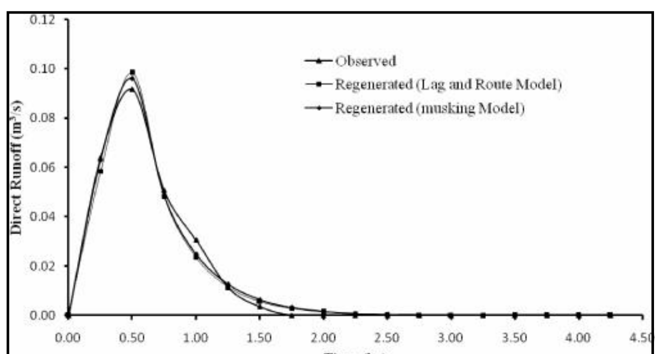

Fig. 1 : Observed and regenerated direct runoff hydrographs through Lag and route and Muskingum model for the storm event of October 9, 2000

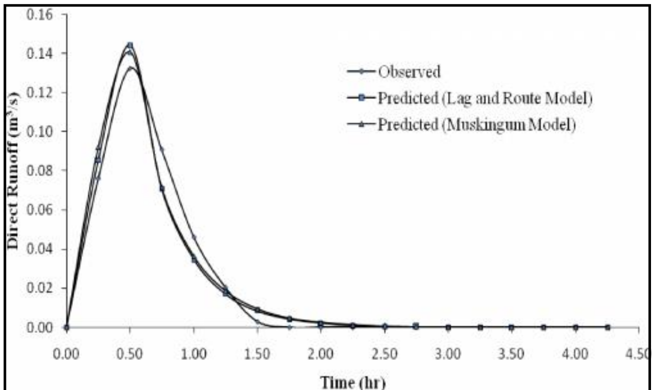

Fig. 2 : Observed and predicted direct runoff hydrographs through Lag and route and Muskingum model for the storm event of June 15, 2004

through Lag and route model were slightly more than that of Muskingum model in both the cases.

From Table 2 it is clear that, the average value of co-efficient of correlation (R) for regenerated and predicted storm event through Lag and route models were 0.96 and 0.96 with an overall average value of 0.96 while that of through Muskingum model were 0.965 and 0.959 with an average value of 0.962, respectively. Sarma *et al.* (1973) reported the ratings of the statistical measures for correlation co-efficient (R) as: $0.99 \leq R < 1.0$ excellent, $0.95 \leq R < 0.99$ very good, $90 \leq R < 0.95$ good, $0.85 \leq R < 0.90$ fair and $0.00 \leq R < 0.85$ poor. Based on the above ratings, both developed models fell under very good category.

It is evident from Table 2 that, average values of

Table 1 : Estimated values of model parameters

Storm event	Lag and route model		Muskingum model
	Storage co-efficient (K) hr	Lag time (τ) hr	Storage constant (K) (hr)
July 12, 2000	0.3723	0.1663	0.46
October 9, 2000	0.3235	0.1470	0.42
June 29, 2005	0.3459	0.1540	0.30
July 4, 2005	0.2492	0.0455	0.32
July 25, 2005	0.3952	0.1676	0.39
August 9, 2005	0.3505	0.1864	0.34
September 8, 2005	0.2959	0.1599	0.38
September 23, 2005	0.3848	0.1941	0.36
July 29, 2006	0.4447	0.1944	0.39
Average value	0.3513	0.1572	0.37

Table 2 : Statistical performance evaluation indices

Storm event	R		R_s		CE		RMSE	
	LR model	M model	LR model	M model	LR model	M model	LR model	M model
July 11-12, 2000	0.989	0.991	0.976	0.979	0.967	0.971	0.0029	0.0007
October 9, 2000	0.994	0.997	0.991	0.995	0.989	0.994	0.0007	0.0005
June 29, 2005	0.993	0.994	0.981	0.989	0.975	0.987	0.0016	0.0009
July 4, 2005	0.809	0.831	0.701	0.736	0.652	0.686	0.0029	0.0016
July 25, 2005	0.979	0.982	0.955	0.958	0.941	0.945	0.0026	0.0028
August 9, 2005	0.974	0.975	0.955	0.955	0.940	0.942	0.0021	0.0021
September 8, 2005	0.991	0.992	0.986	0.987	0.982	0.984	0.0012	0.0012
September 23, 2005	0.962	0.966	0.921	0.926	0.887	0.898	0.0009	0.0008
July 29, 2006	0.951	0.955	0.874	0.881	0.821	0.835	0.0038	0.0035
Average value	0.960	0.965	0.927	0.934	0.927	0.916	0.0021	0.0016
*June 15, 2004	0.985	0.983	0.976	0.975	0.969	0.968	0.0016	0.0016
*July 2, 2006	0.947	0.950	0.873	0.873	0.816	0.820	0.0013	0.0013
*August 9-10, 2008	0.949	0.945	0.912	0.906	0.881	0.875	0.0010	0.0001
Average value	0.960	0.959	0.920	0.918	0.889	0.888	0.0013	0.0010
Total average value	0.960	0.962	0.923	0.926	0.896	0.902	0.0017	0.0013

* Predicted storm events; R =Correlation co-efficient, R_s = Special correlation co-efficient, CE = Co-efficient of efficiency, RMSE =Root mean square error, LR model= Lag and route model, M model = Muskingum model

special correlation co-efficient for Lag and route model for regeneration and prediction purposes were determined to be 0.927 and 0.920 while that of for Muskingum model were determined to be 0.934 and 0.918, respectively. The overall average value of special correlation co-efficient of Lag and route model and Muskingum model were found to be 0.923 and 0.926, respectively. On the basis of ratings reported by Sarma *et al.* (1973) the developed models fell under good category.

The average values of co-efficient of efficiency for Lag and route model for regeneration and prediction purposes were determined to be 0.927 and 0.889 while that of for Muskingum model were determined to be 0.916 and 0.888, respectively. The overall average value of special correlation co-efficient for Lag and route model and Muskingum model were found to be 0.896 and 0.902, respectively. Chiew *et al.* (1993) classified the co-efficient of efficiency into three categories perfectly acceptable simulation (CE > 0.90), acceptable simulation (0.60 < CE < 0.90) and unacceptable simulation (CE < 0.60). On the basis of above criteria developed Lag and route model fell under acceptable simulation category while Muskingum model fell under perfectly acceptable category. The overall average value of root mean square error for the Lag and Route model were determined to be 0.0017 and for Muskingum model was 0.0013which are nearly equal to zero. Hence, the performance of the model is satisfactory for the study watershed.

Conclusion :

From all above statistical criteria used for performance of the developed models, it is clearly seen that both the developed models can be used for prediction of the direct runoff hydrograph from the study watershed, however, direct runoff hydrographs obtained through Muskingum models are much closer to actual observed direct runoff hydrograph than that of Lag and route model.

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