RESEARCH PAPER
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Research Journal of Animal Husbandry and Dairy Science
Volume 9 | Issue 2 | December, 2018 | 26-35

e ISSN-2231-6442
DOI: 10.15740/HAS/RJAHDS/9.2/26-35

Stochastic modeling for milk production

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ABSTRACT : The present investigtion was carried out to study the trends in India's milk production during the period 1990-91 to 2010-2011 based on fuzzy time series and holt-winters non-seasonal time-series models. Model performances have been carried out based on the model performance criteria such as the lower values of mean square error, variability co-efficient (δ) and correlation co-efficient (r) of the model. It was found that the holt-winters non-seasonal time-series was found suitable to study the milk production trend.

KEY WORDS : Window based Fuzzy time series, Mean square error, Variability co-efficient, Holt-Winters non-seasonal model

HOW TO CITE THIS PAPER : Thirunavukkarasu, M.J. and Rajarathinam, A. (2018). Stochastic modeling for milk production. *Res. J. Animal Hus. & Dairy Sci.,* **9**(2) : 26-35 : **DOI: 10.15740/HAS/RJAHDS/9.2/26-35.**

INTRODUCTION

Livestock farming has a increased commercial enterprise in India. The milk production numbers have increased significantly, so has the use of milk by-products. With this increasing demand for milk there rises a need to forecast the milk production value in future. We have been constantly researching in improvising the forecasting techniques to foresee the crop production, drought and femine for example. One cannot make a hundred per cent forecast, but by using efficient statistical tools one can get to a close approximation.

Satya Pal *et al.* (2007) have employed to forecast India's milk production during the period 1980-81 to 2004-05 using the statistical time series modeling techniques – Double Exponential Smoothing and Auto – Regressive Integrated Moving Average (ARIMA) models. The time series models often explain the current values of the dependent variables as functions of past values of the dependent and independent variables. These past values are referred to as lagged values, and the variables x_{t-i} is called lag i of the variable x_i. If the data are time series, so that t indexes time, it is possible that e_t, the error of the model at time t, depends on e_{t-i} or, more generally, the e_t's are not identically and independently distributed. If the errors of a model are autocorrelated, the standard error of the estimates of the parameters of the system will be inflated. Some times the e_t's are not identically distributed because the variance of e is not constant. This is known as *Heteroscedasticity* which inflate the standard error of the estimates of the parameters of the model.

Looking to the above drawbacks, fuzzy time series and holt-winter non-seasonal models have been employed to the trends in India's milk production data set.

MATERIAL AND METHODS

In order to achieve the stipulated objectives of the present investigation time-seies data on Indian milk production pertaining to the period 1990-91 to 2010-2011 have been collected through the Department of Animal Husbandry, Dairying and Fisheries, Ministry of Agriculture, Government Agricultre Statistical office in Chennai, Tamil

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Nadu, India. The Fuzzy time series model and Holt-Winters non-seasonal model have been employed to study the milk production trends and the model performances have been studied based on the model performance criteria such as the lower values of mean square error (MSE), variability co-efficient (δ) and correlation co-efficient (r) of the model.

In order to study the trends in India's milk production the window based fuzzy time series models (Jeng-Ren Hwang *et al.*, 1998) and holt-winters non-seasonal time-series models have been employed. The details of these procedures are discussed below.

Window based fuzzy time series (Jeng-Ren Hwang *et al.*, 1998) :

Assume that the milk production of year t is x and assume that the milk production of year t – 1 is y, then the variation of the milk production between year t and year t–1 is equal to x–y. Firstly, we describe some heuristic rules which are similar to the human thought:

Rule 1:

The variation of the milk production between this year and last year is related to the variations of the milk production between this year and the past years, and the relationship of the milk production between this year and last year is closer than the one between this year and the other past years.

Rule 2:

If the trend of the number of milk production of the past years is increasing, then the number of milk production of this year is increasing. If the trend of the number of milk production of the past years is decreasing, then the number of milk production of this year is decreasing.

From rules 1 and 2, we might have two problems. Firstly, if the trend of the variations of the milk production of the past years is not so obvious, how can we know the trend of the variation of the milk production this year? Secondly, how to define the degree of variation of this year? The solutions of these two problems are described by the following heuristic rule:

Rule 3:

Let the variation of last year be a criterion. Compute the fuzzy relationships between last year and the other past years based on data variations. From the derived fuzzy relationships, one can know the degrees of relationships between the variation of last year and the variations of other past years. The variation of this year can be obtained from the derived fuzzy relationships.

Based on these heuristic rules, firstly we can fuzzify the milk production data. Sullivan and Woodall (1994) and Jeng-Ren Hwang *et al.* (1998) used the linguistic values (not many), (not too many), (many), (many many), (very many), (too many), (too many many) to forecast the enrollments of the University of Alabama. In this investigation, the fuzzified variation of the historical milk production and the linguistic values *viz.*, (big decrease), (decrease), (no change), (increase), (big increase), (too big increase) have been used to forecast the milk production of India. The fuzzified variation of the historical milk production between year t and year t – 1 are described below.

$$\mathbf{F\ (t) = u_1\ /(big\ decrease) + u_2\ /(decrease) + \dots + u_i/(L) + \dots + u_m\ /\ (\ too\ big\ increase)} \quad \dots(1)$$

where, F (t) denotes the fuzzified variation of the milk production between year t and year t – 1, u_i is the grade of membership to the linguistic value L, m is the number of the elements in the universe of discourse, and $1 \leq i \leq m$.

To forecast the milk production of year t, one must decide how many years of the milk production data will be used,where the number of years of the milk production data we used is called the window basis. Suppose we set a window basis to w years, then the variation of last year is used to be a criterion and the other variations of w past years are used to form a matrix which is called the operation matrix. The criterion matrix C (t) and the operation matrix O^w (t) at year t are given below :

$$C(t) = F\ (t-1) = \begin{matrix} \text{(bigdecrease)} & \text{(decrease)} & \dots & \text{(too big increase)} \\ [\ C_1 & C_2 & \dots & C_m\ ] \end{matrix} \quad \dots(2)$$

$$O^w\ (t) = \begin{bmatrix} F(t-2) \\ F(t-3) \\ \vdots \\ F(t-w-1) \end{bmatrix} = \begin{matrix} \text{(big decrease)} & \text{(decrease)} & \dots & \text{(too big increase)} \\ \end{matrix} \begin{bmatrix} O_{11} & O_{12} & \dots & O_{1m} \\ O_{21} & O_{22} & \dots & O_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ O_{W1} & O_{w2} & \dots & O_{wm} \end{bmatrix} \quad \dots(3)$$

One can calculate the relation between the operation matrix O^w (t) and the criterion matrix C(t), and get a relation matrix R(t)[w, m] by performing $R(t) = O^w\ (t) \otimes C(t)$, where

$$R(t) = \begin{bmatrix} O_{11} \times C_1 & O_{12} \times C_2 \cdots O_{1m} \times C_m \\ O_{21} \times C_1 & O_{22} \times C_2 \cdots O_{2m} \times C_m \\ \vdots & \vdots \\ O_{w1} \times C_1 & O_{w2} \times C_2 \cdots O_{wm} \times C_m \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} \cdots & R_{1m} \\ R_{21} & R_{22} \cdots & R_{2m} \\ \vdots & \vdots & \vdots \\ R_{W1} & R_{w2} \cdots & R_{wm} \end{bmatrix} \quad ..(4)$$

where $R_{ij} = O_{ij} \times C_j$, $1 \leq i \leq w$, $1 \leq j \leq m$, and "×" is the multiplication operation. From the relation matrix R(t), we can know the degree of relationships between last year and the other past years in data variations. Then, we can get the forecasting variation of the milk production data of year t, where

$$F(t) = [\ Max(R_{11}, R_{21}, \ldots, R_{w1}) \ \ldots \ Max(R_{12}, R_{22}, \ldots, R_{w2}) \ \ldots \ Max(R_{1m}, R_{2m}, \ldots, R_{wm})] \quad \ldots(5)$$

Holt-winters non-seasonal model (Liu Cuicui and Yun Jun, 2012) :

The holt-winters non-seasonal method is one kind of time series analysis and forecast method, which combined with a certain amount of time series prediction the future by calculating the exponential smoothing value. In extended single exponential smoothing to linear exponential smoothing to allow forecasting data with trends. Time series data of India milk production has trend but non seasonal trends, so select the appropriate holt-winters non seasonal model to prediction future. This method is found using two smoothing constants, α and β, and make compute following three eq.:

$$L_t = \sim Y_t + (1 - \sim)(L_{t-1} + b_{t-1})$$
$$b_t = \quad (L_t - L_{t-1}) + (1 - \quad) b_{t-1}$$
$$F_{t+k} = L_t + b_t k$$

where L_t is the intercept, and b_t is the slope and $k>0$, $\alpha=1$, $\beta=1$ is smoothing constant. So the smoothing constant giving larger weights to recent observations and smaller weights to long-term observation. Predictive values are calculated by $F_{t+k} = L_t + b_t k$.

Model performance measures :

Model performance have been carried out based on accuracy of the different forecast models. Accuracy refers to the extent consistent with the predicted result with the actual situation, which can be reflected by the error indicators.

$x_1, x_2, \ldots, x_n$ is observation data. The corresponding prediction value is $\hat{x}_1, \hat{x}_2, \ldots\ldots\ldots, \hat{x}_n$.

Mean square error(MSE) is

$$MSE = \frac{1}{n} \sum_{i=1}^{n} (x_i - \hat{x}_i)^2$$

Variability co-efficient (δ) is

$$\delta = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_i - \hat{x}_i)^2} \bullet \frac{1}{\frac{\sum_{i=1}^{n} \hat{x}_i}{n}}$$

Correlation co-efficient (R) is

$$R = 1 - \sqrt{\frac{\sum_{i=1}^{n} (x_i - \hat{x}_i)^2}{\sum_{i=1}^{n} x_i^2}}$$

RESULTS AND DISCUSSION

Window based fuzzy time series and holt-winters non-seasonal models have been employed to study the milk production trend and the results are discussed in detail in the following sections.

The window based fuzzy time series is now presented as follows:

Step 1:

From the historical milk production data, compute the variations in milk production the milk data between any two consecutive years. For example, if the milk production in 1992 is 58 and the milk production in the previous year 1991 is 55.6, then the variation of years 1992 = 58 – 55.6 = 2.4. Based on the historical milk production data, the variations of the milk data between any two continuous years are given in Table 1. From this one can find the minimum increase D_{min} and maximum increase D_{max}. Then define the universe of discourse U, $U = [D_{min} - D_1, D_{max} + D_2]$, where D_1 and D_2 are suitable positive numbers. Here set $D_{min} = 1.8$, $D_{max} = 6.1$, $D_1 = -1.8$, $D_2 = 0.4$, so U can be represented as U =[0, 6.5].

Step 2:

Partition the universe of discourse U into several even length intervals $u_1, u_2, \ldots, u_m$. Partition the universe of discourse U into six intervals, where $u_1 = [0, 1.1]$, $u_2 = [1.1, 2.2]$, $u_3 = [2.2, 3.3]$, $u_4 = [3.3, 4.4]$, $u_5 = [4.4, 5.5]$, and $u_6 = [5.5, 6.6]$.

Step 3:

Define fuzzy sets on the universe of discourse U. First, determine some linguistic values represented by fuzzy sets to describe the degree of variations between any two consecutive years. In this investigation, six fuzzy sets which are A_1= (big decrease), A_2= (decrease), A_3= (no change), A_4= (increase), A_5= (big increase), A_6= (too big increase) have been considered. Then, define fuzzy sets $A_1, A_2, \ldots, A_6$ on the universe of discourse U as follows :

$A_1 = 1/u_1 + 0.5/u_2 + 0/u_3 + 0/u_4 + 0/u_5 + 0/u_6$,
$A_2 = 0.5/u_1 + 1/u_2 + 0.5/u_3 + 0/u_4 + 0/u_5 + 0/u_6$,
$A_3 = 0/u_1 + 0.5/u_2 + 1/u_3 + 0.5/u_4 + 0/u_5 + 0/u_6$,
$A_4 = 0/u_1 + 0/u_2 + 0.5/u_3 + 1/u_4 + 0.5/u_5 + 0/u_6$,
$A_5 = 0/u_1 + 0/u_2 + 0/u_3 + 0.5/u_4 + 1/u_5 + 0.5/u_6$,
$A_6 = 0/u_1 + 0/u_2 + 0/u_3 + 0/u_4 + 0.5/u_5 + 1/u_6$.

Step 4:

Fuzzify the values of historical milk data. If the

Table 1 : Milk production and variations of historical data

Year	Milk production	Variation
1991	55.6	
1992	58.0	2.4
1993	60.6	2.6
1994	63.8	3.2
1995	66.2	2.4
1996	69.1	2.9
1997	72.1	3.0
1998	75.4	3.3
1999	78.3	2.9
2000	80.6	2.3
2001	84.4	3.8
2002	86.2	1.8
2003	88.1	1.9
2004	92.5	4.4
2005	97.1	4.6
2006	102.6	5.5
2007	107.9	5.3
2008	112.2	4.3
2009	116.4	4.2
2010	121.8	5.4
2011	127.9	6.1

Table 2: Fuzzified milk production in India

Year	Milk production	Fuzzified variations
1991		
1992	2.4	A_3
1993	2.6	A_3
1994	3.2	A_3
1995	2.4	A_3
1996	2.9	A_3
1997	3.0	A_3
1998	3.3	A_4
1999	2.9	A_3
2000	2.3	A_3
2001	3.8	A_4
2002	1.8	A_2
2003	1.9	A_2
2004	4.4	A_5
2005	4.6	A_5
2006	5.5	A_6
2007	5.3	A_6
2008	4.3	A_5
2009	4.2	A_4
2010	5.4	A_5
2011	6.1	A_6

number of variation of the milk data of year i is p where $p \in u_j$, and if there is a value represented by fuzzy set A_k in which the maximum membership value occurs at u_j, then p is translated to A_k. The fuzzified variations of the milk data is given in the Table 2.

Step 5:

Choose a suitable window basis O^w (t) and calculated the output from the operation matrix and the criterion matrix C(t), where t is the year for which we want to forecast value is needed. For example, if we set w = 5, then we can set a 4 × 6 operation matrix O^5 (t) and a 1 × 6 criterion matrix C(t). Because w = 5, we must use six past years milk data, so we begin to forecast in 1997. In this case, the operation matrix O^5 (t) and the criterion matrix C(t) are as follows:

$$O^5(1997) = \begin{bmatrix} \text{Fuzzy variation of the milk production of 1995} \\ \text{Fuzzy variation of the milk production of 1994} \\ \text{Fuzzy variation of the milk production of 1993} \\ \text{Fuzzy variation of the milk production of 1992} \end{bmatrix} = \begin{bmatrix} A_3 \\ A_3 \\ A_3 \\ A_3 \end{bmatrix}$$

(Big decrease)(Decrease)(No change)(Increase)(Big increase)(Too big increase)

$$= \begin{bmatrix} 0 & 0.5 & 1 & 0.5 & 0 & 0 \\ 0 & 0.5 & 1 & 0.5 & 0 & 0 \\ 0 & 0.5 & 1 & 0.5 & 0 & 0 \\ 0 & 0.5 & 1 & 0.5 & 0 & 0 \end{bmatrix}$$

C (1997) = fuzzy variation of the milk production of 1996 = [A_3]

(Big decrease) (decrease) (no change) (increase) (big increase) (too big increase)

$$= [\; 0 \quad 0.5 \quad 1 \quad 0.5 \quad 0 \quad 0 \;]$$

Calculate the relation matrix R(t) by R(t)[i, j] = O^w (t) [i, j] × C(t)[j], where $1 \le i \le 4$, and $1 \le j \le 6$. Then, based on formula (3.4), we can get

(Big decrease) (decrease) (no change) (increase) (big increase) (too big increase)

$$R(1997) = \begin{bmatrix} 0 & 0.25 & 1 & 0.25 & 0 & 0 \\ 0 & 0.25 & 1 & 0.25 & 0 & 0 \\ 0 & 0.25 & 1 & 0.25 & 0 & 0 \\ 0 & 0.25 & 1 & 0.25 & 0 & 0 \end{bmatrix}$$

Based on formula (5), one can get the fuzzified forecasting variation F(1997) of year 1997 shown as follows:

(Big decrease) (Decrease) (No change) (Increase) (Big increase) (Too big increase)

$$F(1997) = [0 \quad 0.25 \quad 1 \quad 0.25 \quad 0 \quad 0]$$

The fuzzified forecasted variations for the remaining years can be calculated by the same way and all the results are given in the Table 3.

Step 6:

Defuzzify the fuzzy forecasted variations derived in Step 5. In this paper, we use the following principles to defuzzify the fuzzified forecasted variations:

– If the grades of membership of the fuzzified forecasted variation have only one maximum u_i, and the midpoint of u_i is m_i, then the forecasted variation is m_i. If the grades of membership of the fuzzified forecasted variation have more than one maximum $u_1, u_2, ..., u_k$ and their midpoints are $m_1, m_2, ..., m_k$, respectively, then the forecasted variation is $(m_1 + m_2 + . . . + m_k)/k$. For

Table 3 : Forecasted variations with the window basis w = 5

Years	Membership function of forecasted variations					
	u_1	u_2	u_3	u_4	u_5	u_6
1997	0	0.25	1	0.25	0	0
1998	0	0.25	1	0.25	0	0
1999	0	0	0.5	0.5	0	0
2000	0	0.25	1	0.5	0	0
2001	0	0.25	1	0.5	0	0
2002	0	0	0.5	1	0.25	0
2003	0	0.5	0.5	0	0	0
2004	0.25	1	0.5	0	0	0
2005	0	0	0	0.5	0.5	0
2006	0	0	0	0.5	1	0.25
2007	0	0	0	0	0.5	0.5
2008	0	0	0	0	0.5	1
2009	0	0	0	0.25	1	0.5
2010	0	0	0	0.5	0.5	0
2011	0	0	0	0.5	1	0.5
2012	0	0	0	0	0.5	1

example, from Table 3, we can see that the maximum membership value of F(1997) is 1 which occurs at u_3, where the midpoint of u_3 is 2.75. The forecasted variation of 1997 is 2.75.

– If the grades of membership of the fuzzified forecasted variation are all 0, then we set the forecasted variation to 0.

Step 7:

Calculate the forecasted milk production data. The

Table 4 : Forecasted results using the fuzzy time-series method with the window basis w= 5

Year	Observed milk production	Forecasted milk production	Errors (%)
1997	72.1	71.9	0.35
1998	75.4	74.9	0.73
1999	78.3	78.7	0.51
2000	80.6	81.1	0.56
2001	84.4	83.4	1.26
2002	86.2	88.3	2.32
2003	88.1	88.4	0.34
2004	92.5	89.8	3.06
2005	97.1	96.9	0.21
2006	102.6	102.1	0.54
2007	107.9	108.1	0.19
2008	112.2	113.9	1.54
2009	116.4	117.2	0.64
2010	121.8	120.8	0.83
2011	127.9	126.8	0.91
2012	Forecast	133.9	

Table 5 : Forecasting milk production with different window bases

Year	Actual milk data	Forecasted milk production							
		W=2	W=3	W=4	W=5	W=6	W=7	W=8	W=9
1994	63.8	63.4							
1995	66.2	66.6	66.6						
1996	69.1	69.0	69.0	69.0					
1997	72.1	72.0	71.9	71.9	71.9				
1998	75.4	74.9	74.9	74.9	74.9	74.9			
1999	78.3	78.7	78.7	78.7	78.7	78.7	78.7		
2000	80.6	81.6	81.1	81.1	81.1	81.1	81.5	81.1	
2001	84.4	83.4	83.4	83.4	83.4	83.4	84.5	83.4	83.4
2002	86.2	87.7	87.7	88.3	88.3	87.7	88.3	88.3	88.3
2003	88.1	89.0	84.4	90.1	88.4	84.4	88.4	88.4	88.4
2004	92.5	89.8	89.8	89.8	89.8	89.8	89.8	89.8	89.8
2005	97.1	92.5	92.5	96.9	96.9	96.9	96.9	96.9	96.9
2006	102.6	101.1	102.1	102.1	102.1	102.1	102.1	102.1	102.1
2007	107.9	108.1	108.1	108.1	108.1	108.1	108.1	108.7	108.1
2008	112.2	114.0	114.0	114.0	114.0	114.0	114.0	114.0	114.0
2009	116.4	117.7	118.3	117.2	117.2	117.2	116.6	117.2	117.2
2010	121.8	120.8	120.8	120.8	120.8	120.8	120.8	120.8	120.3
2011	127.9	126.8	126.8	126.8	126.8	126.8	126.8	126.8	126.8
2012	Forecast	134.4	133.4	133.4	134.0	134.0	132.9	134.0	134.0

forecasted milk production is forecasted variation plus the number of actual milk production of last year. For example, if the forecasted in 1997 is 2.75, and the actual milk production in 1996 is 69.1, then the forecasted milk data of 1997 is 69.1 + 2.75 = 71.85. The results of the forecasted milk production of the India are shown in Table 4. The following error of each year by the fuzzy time series method under the window basis w = 5 is also shown in Table 4.

Empirical analysis :

Table 5 shows the forecasting results of different window bases ranging from 2 to 9. From Table 6 we can see that the average forecasting errors for different window bases range from 1.38 per cent down to 0.86 per cent. From Table 6, we can see that the biggest average forecasting error (1.38 %) occurred at w=3, and the smallest forecasting error (0.86 %) occurred at w=7. It is difficult to find the relationships between the window basis and the average forecasting error, but there is an efficient way (Song and Chissom, 1993) which uses genetic algorithms to find the better window basis used to forecast. Where the different window basis aveage forecasting error value, so choose window w=3 the biggest average forecasting error based forecasting the fuzzy time series model.

The curve of the actual milk production and the forecasted milk productions are shown in Fig. 1, where

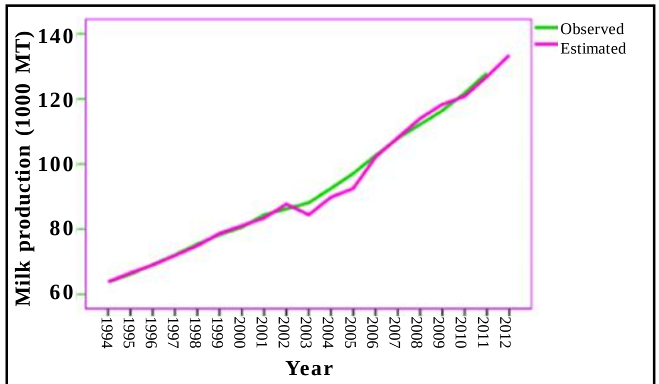

Fig. 1 : Forecast of Fuzzy time series window base w = 3

Table 6 : Forecasting errors with different window bases

	Window bases							
	W=2	W=3	W=4	W=5	W=6	W=7	W=8	W=9
Average forecasting errors	1.19%	1.38%	1%	0.93%	1.20%	0.86%	1.07%	1.11%

Table 7 : Forecasting milk production with different smoothing constant bases

Year	Actual milk data	Forecasted milk production									
		α =1 β=0.6	α =1 β=0.7	α =1 β=0.8	α =1 β=0.3	α =1 β=0.9	α =1 β=0.4	α=0.9 β=0.8	α=0.9 β=0.9	α=0.9 β=0.7	α=1 β=1
1992	58.0	58.0	58.0	58.0	58.0	58.0	58.0	58.0	58.0	58.0	58.0
1993	60.6	60.4	60.4	60.4	60.4	60.4	60.4	60.4	60.4	60.4	60.4
1994	63.8	63.1	63.1	63.2	63.1	63.2	63.1	63.1	63.1	63.1	63.2
1995	66.2	66.7	66.8	66.9	66.5	66.9	66.6	66.8	66.8	66.7	67.0
1996	69.1	68.8	68.8	68.7	68.8	68.7	68.8	68.9	68.9	68.9	68.6
1997	72.1	71.9	71.9	71.9	71.8	72	71.8	71.9	71.9	71.9	72.0
1998	75.4	75.0	75.0	75.1	74.9	75.1	74.9	75.0	75.1	75.0	75.1
1999	78.3	78.6	78.6	78.6	78.3	78.7	78.4	78.6	78.6	78.5	78.7
2000	80.6	81.3	81.3	81.3	81.2	81.2	81.3	81.3	81.3	81.4	81.2
2001	84.4	83.2	83.1	83.0	83.3	83	83.3	83.2	83.1	83.2	82.9
2002	86.2	87.7	87.8	87.9	87.5	88.1	87.5	87.7	87.7	87.6	88.2
2003	88.1	88.6	88.5	88.4	88.9	88.2	88.8	88.7	88.6	88.8	88.0
2004	92.5	90.2	90.1	90.1	90.6	90	90.4	90.1	90	90.2	90.0
2005	97.1	96.0	96.2	96.4	95.5	96.7	95.7	95.9	96.1	95.7	96.9
2006	102.6	101.3	101.4	101.6	100.6	101.7	100.8	101.5	101.7	101.3	101.7
2007	107.9	107.6	107.8	107.9	106.7	108	107	107.8	107.9	107.6	108.1
2008	112.2	113.1	113.2	113.2	112.4	113.2	112.7	113.3	113.3	113.2	113.2
2009	116.4	116.9	116.8	116.7	115.6	116.6	116.8	116.9	116.8	117	116.5
2010	120.8	120.7	120.7	120.7	120.8	120.6	120.8	120.7	120.6	120.8	120.6
2011	127.9	126.8	126.9	127.0	126.5	127.1	126.6	126.7	126.8	126.7	127.2

the window basis is 3. Table 7 shows the forecasting results of different smoothing constant bases ranging from 0 to 1. In the Holt-Winters non-seasonal model the parameter values for α and β have been choosen so that there is a minimum average error. From Table 8 it is appearing that the average forecasting error ranges from 0.78 % (α=1, β=1) to 0.87 % (α =1, β=0.3).

The following figure depicts the India's milk production trend based on the maximum average forecasting error values 0.87 % (α =1, β=0.3) using the holt – winter non-seasonal model.

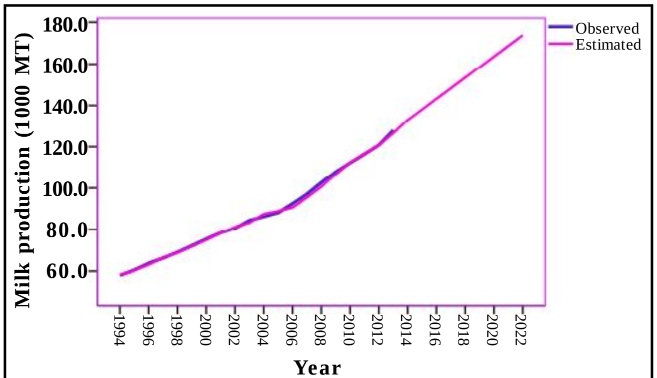

Fig. 2 : Trends in milk production based on holt-winter non-seasonal model

Comparison of forecast models :

The Fig. 3 depicts trends in India's milk production based on fuzzy times series as well as holt-winters non-seasonal models. The prediction results for both model holds good only for short range forecast, comapritively model HW has lower error value but the same increases with increase in range. The per cent error in 2011 is beyond 1.1 per cent. HW Model is predictive and is always lower than the actual value however, the percentage error is lower. When it comes to model stability, for the fitting results, there are subtle differences

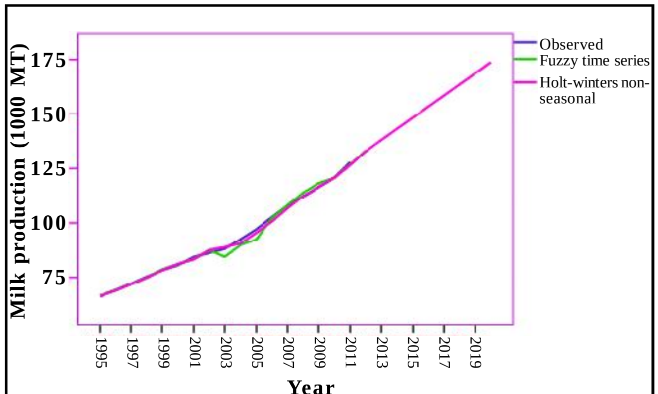

Fig. 3 : Comparison of fuzzy time series and holt-winter non-seasonal model

Table 8 : Forecasting errors with different smoothing constant bases

	Smoothing constant bases									
	α =1 β=0.6	α =1 β=0.7	α =1 β=0.8	α =1 β=0.3	α =1 β=0.9	α =1 β=0.4	α =0.9 β=0.8	α =0.9 β=0.9	α=0.9 β=0.7	α=1 β=1
Average forecasting errors	0.83%	0.83%	0.80%	0.87%	0.79%	0.84%	0.84%	0.83%	0.85%	0.7%

Table 9 : Observed and forecast value of the two model with error percentage

Milk production in India		Fuzzy time series model		Holt-Winters non-seasonal model (HW)	
Year	Observed	Forecast	Error (%)	Forecast	Error (%)
2009	116.4	118.3	1.63	115.6	0.68
2010	121.8	120.8	0.82	120.8	0.82
2011	127.9	126.8	0.86	126.5	1.09

Table 10 : India milk production of holt-winters non- seasonal prediction models (in Million Tonnes)

Year	2012	2013	2014	2015	2017	2020
Forecast	133	138.1	143.2	148.3	158.5	173.8

Table 11 : Error indicators of both models

Error indicator	MSE	δ	R
Fuzzy time series window basis W=3	3.28160	0.01951	0.9809
Holt-winters non-seasonal model α =1 and β=0.3	1.01301	0.01143	0.9889

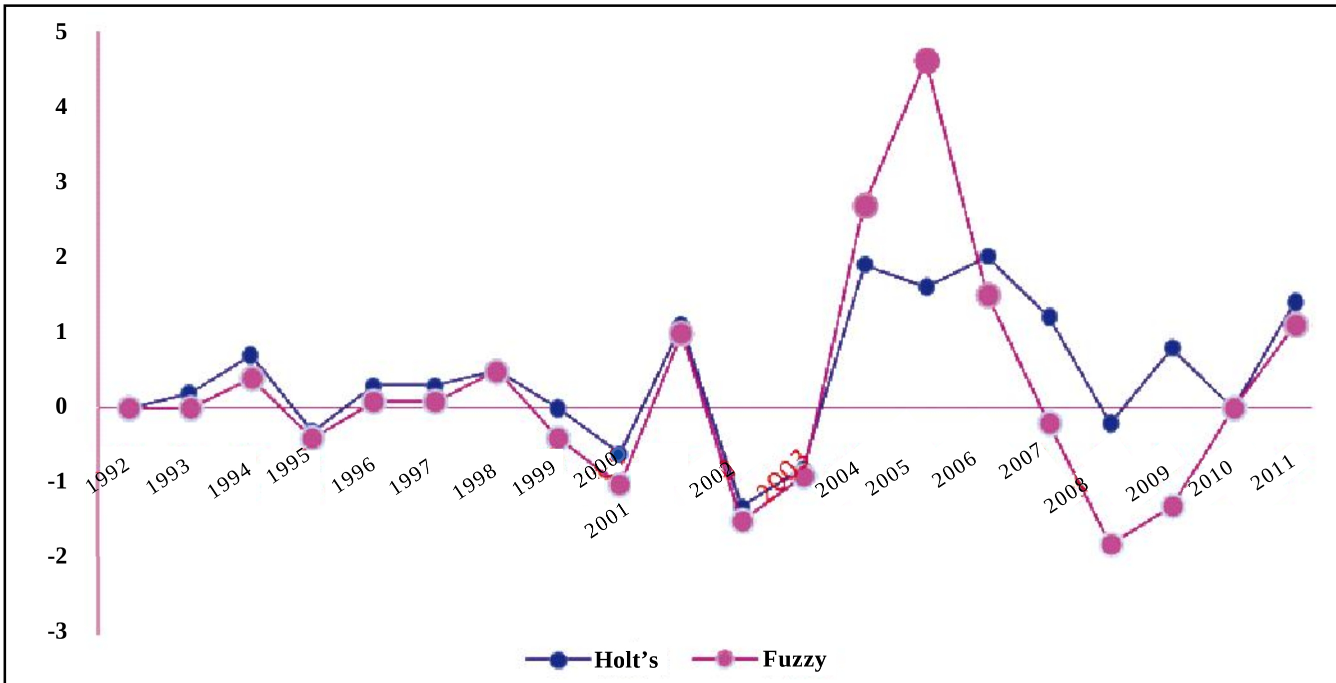

Fig. 4 : Error comparison of fuzzy time series and holt-winter non-seasonal models

between models which is difficult to be distinguished. So graphically compare both models in Fig. 4.

Although the percentage error of model one is within 1.1 per cent it is always lower than the actual value. Based on the rapid development of India's milk production and the growth of milk production will continue to expand. Thus we define the correction factor:

a=average (-0.0068, -0.0082, -0.0109) = -0.00863≈ -0.009. The modified results are shown below:

Using the data for output value of milk production in India from 1991 to 2011, we modeled this data using fuzzy time series and holt-winters non-seasonal model. We then analyzed and compared both model. The holt-winters non-seasonal model was the best fit. Based on this model, we conducted concervative prediction values.

The perusal of the Table 11 reveals that the MSE of holt-winters non-seasonal model (1.01301) is smaller than that of fuzzy time series (3.28160). δ of holt-winters non-seasonal model (0.01143) is smaller than that of fuzzy time series (0.01951). R of holt-winters non-seasonal model (0.9889) is larger than that of fuzzy time series (0.9809). Therefore, relative to the selected fuzzy time series model, the holt-winters non-seasonal model is more applicable to trend of India's milk production.

Conclusion :

In this investigation, both the forecasting models can be used to study the trends in India's milk production. The Holt-Winters Non-seasonal model clearly reflects the volatility of milk production trend and simultaneously maintained the stability of forecast. The prediction error of Holt-Winters non- seasonal model was significantly lower compared to the Fuzzy time series model. Thus, it can be suitably concluded that both the models proved suitable for the actual forecast of milk production trends since they have subtle differences in their respective prediction values, where the Holt-Winters non-seasonal model has an upper edge.

Acknowledgements :

The first author expresses his gratitude to the UGC for providing fellowship to carry out this work under scheme of BSR- Fellowship.

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***Received :** 21.08.2018; **Revised:** 05.11.2018; **Accepted :** 14.11.2018*